IASPEI
10 - 16 JAN 2009
H2: Workshop "Geothermal Studies: Instruments, Measurements, and Interpretation

Geostatistical approaches on the thermal conductivities of rocks

Byoung Ohan Shim, Jeongmin Park, Hyoung Chan Kim, Youngmin Lee

Korea Institute of Geoscience and Mineral Resources, Republic of Korea,
boshim@kigam.re.kr

Objective and background

- **Interpretation of the measured thermal properties of rocks in KIGAM geothermal D/B system**

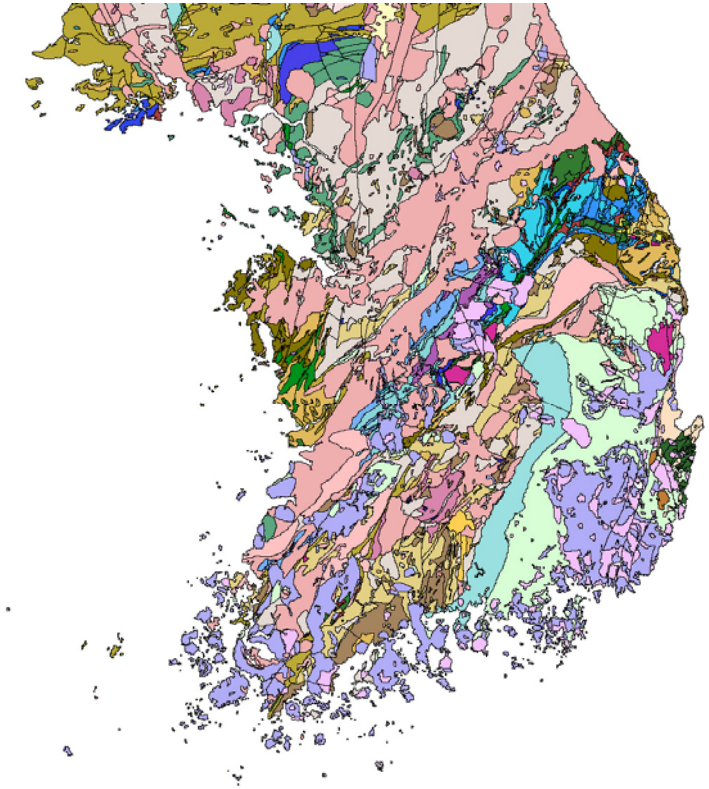
- **Statistical investigation and geostatistical analysis of the thermal conductivity of rocks**

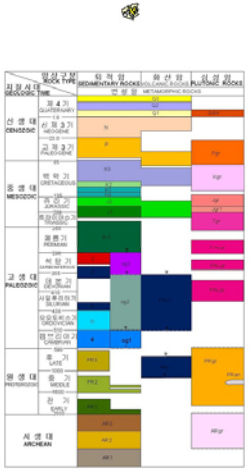
- **Demands of thermal conductivity data have been increased from geothermal heat pump industries**

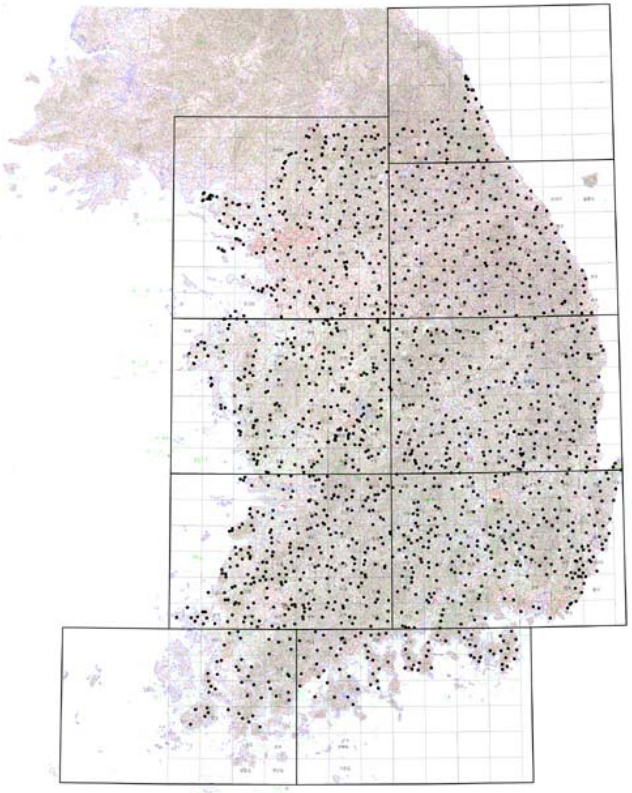
From geostatistical analysis on the thermal conductivity of rocks

- **Characterization of the spatial structure of collected data**
- **To increase the reliability of estimated data**
- **Generation of the best thermal conductivity distribution maps**

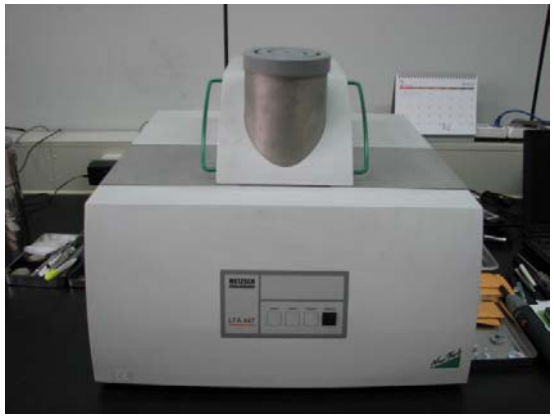
Geological map of the Rep. of Korea



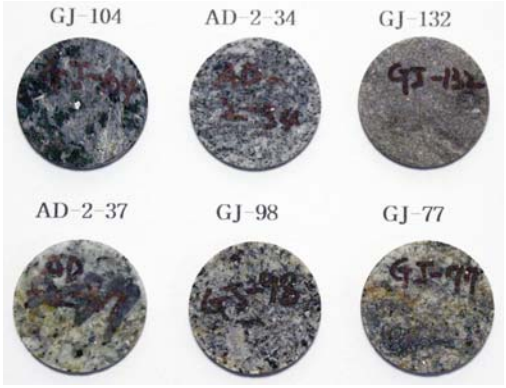




Measurements of rock properties

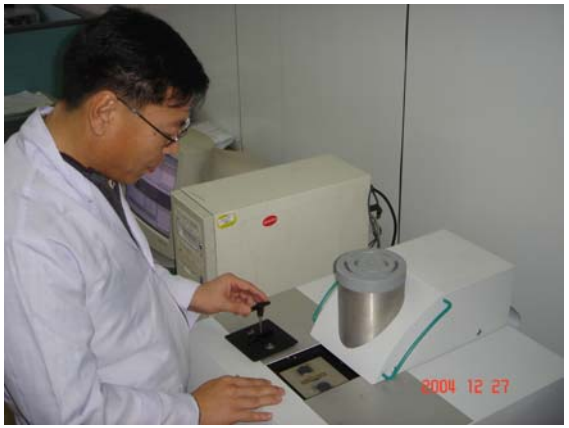


Thermal diffusivity

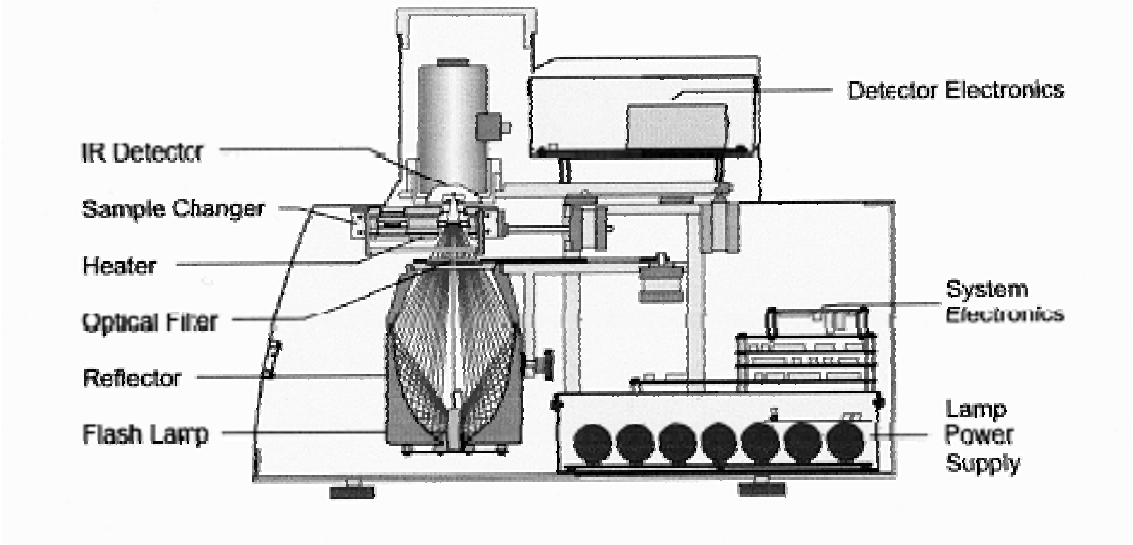


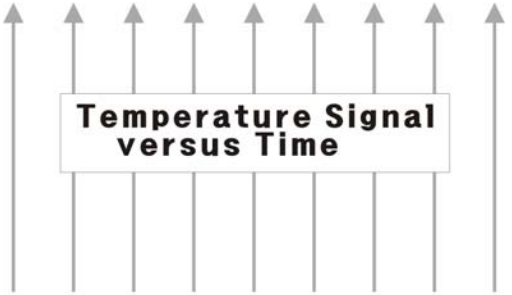
density, porosity,
specific heat,
thermal diffusivity,
thermal conductivity
(dry, saturated):
1560 specimens

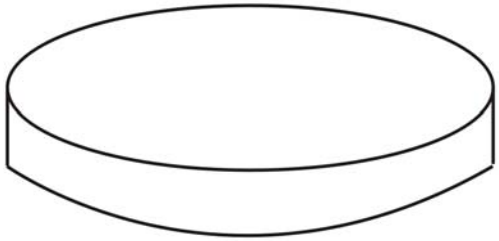
Density

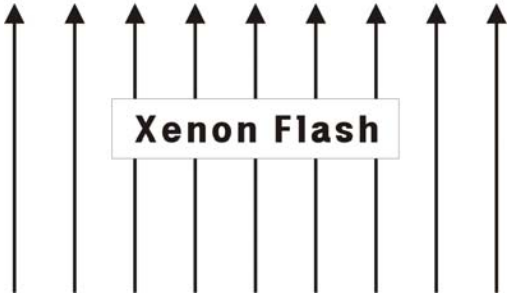


Principle of flash method (LFA 447)









$$\lambda = \alpha \rho C_p$$

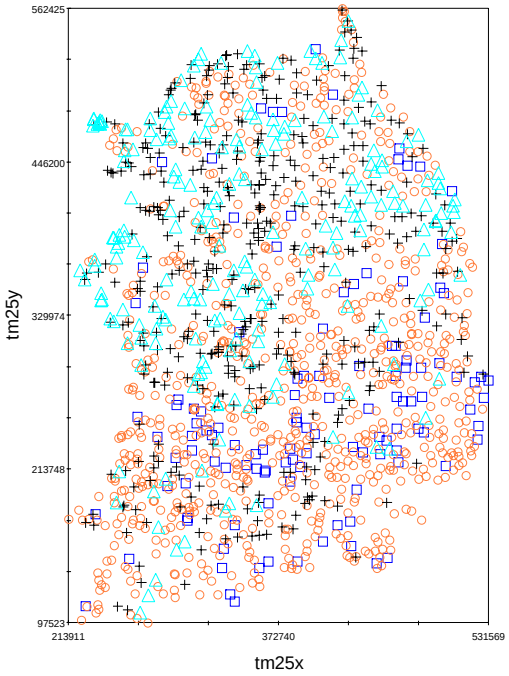
λ : **Thermal diffusivity**

α : **Thermal conductivity**

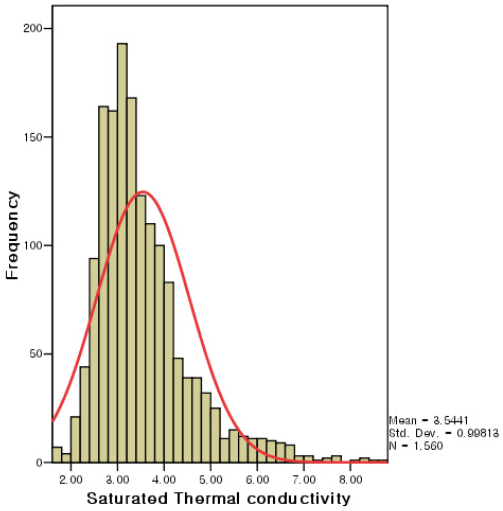
ρ : **Density**

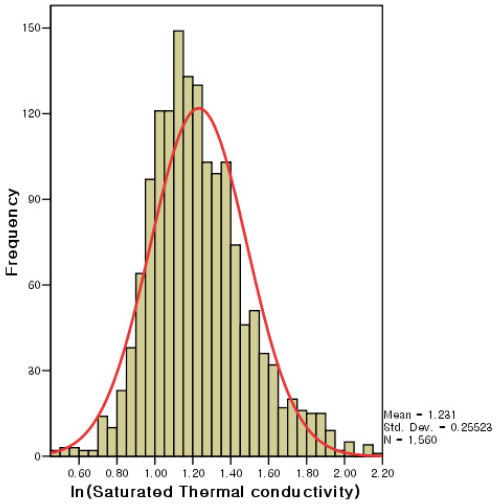
Cp : **Specific heat**

Thermal conductivity distribution of total samples



Saturated Thermal conductivity
- below 2.50
- 2.5 - 3.50
- 3.5 - 4.50
- 4.5 - 8.76 (max)





Total : 1560 ea

Igneous rocks: 647 ea

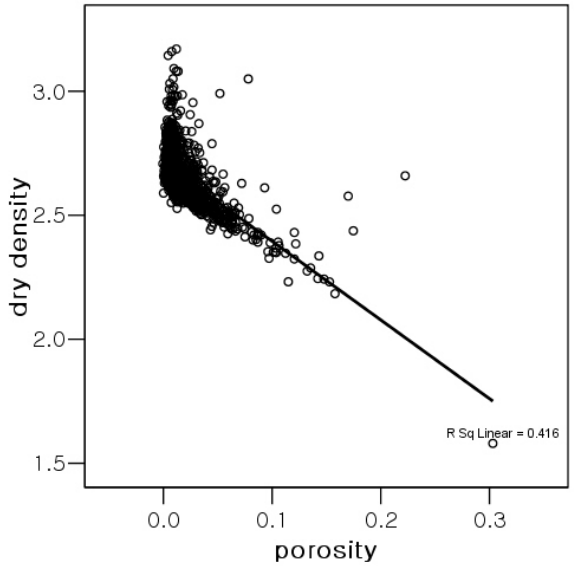
Metamorphic rocks: 494 ea

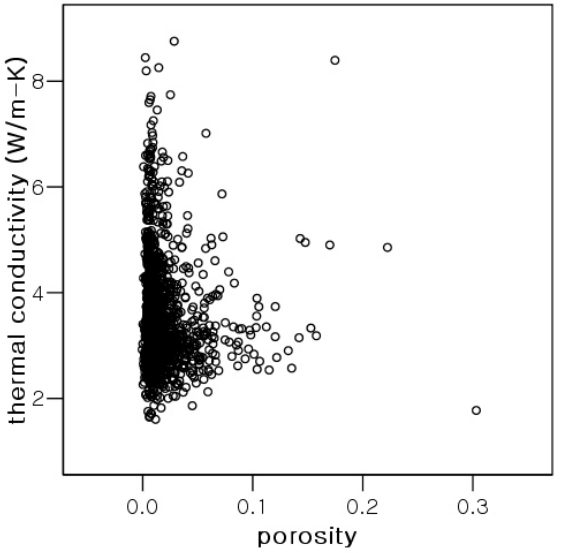
Sedimentary rocks: 291 ea

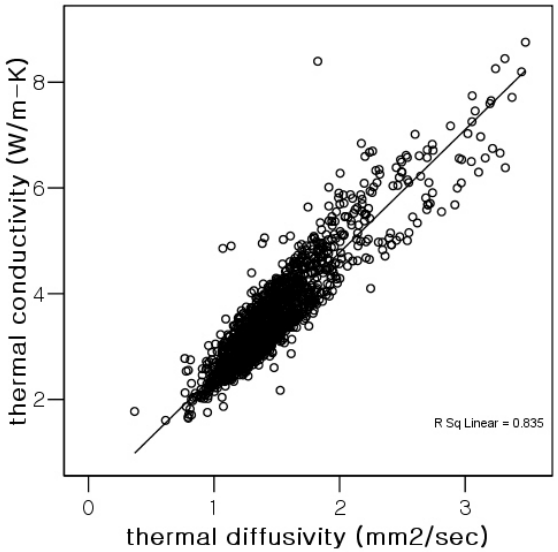
Volcanic rocks: 128

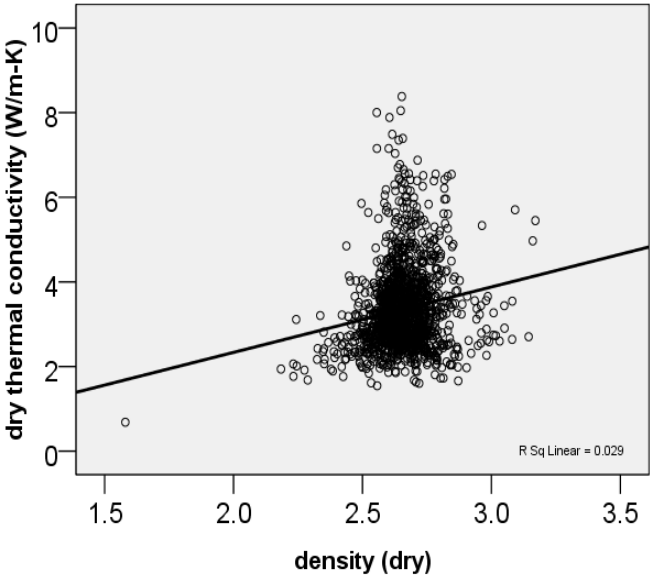
statistics	Non Transformed	Log Transformed
minimum	1.6	
maximum	8.76	
Mean (W/m-K)	3.542	3.423
SD	0.996	1.033
Variance	0.99234	1.06710
Skewness	1.53	0.60
Kurtosis	3.82	0.66

relationships between property variables

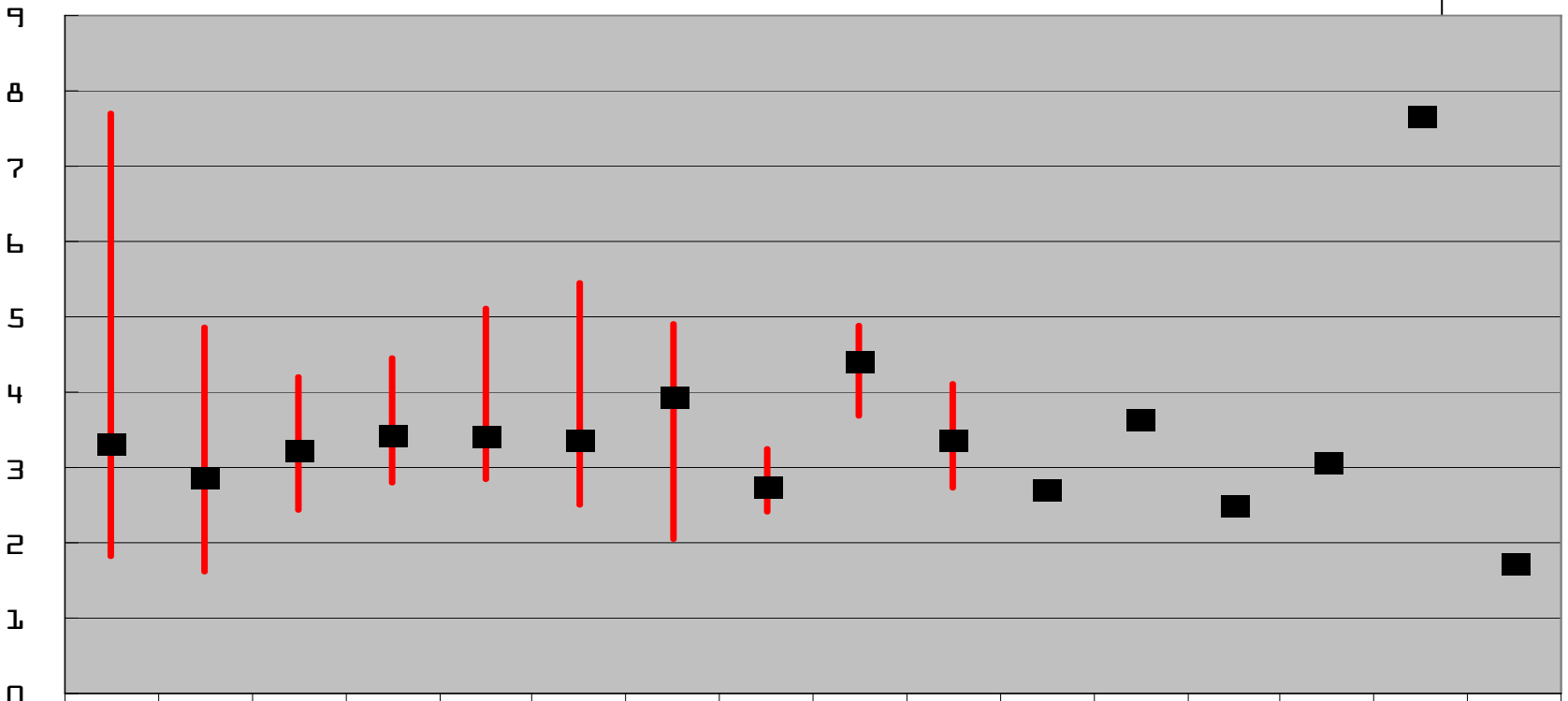






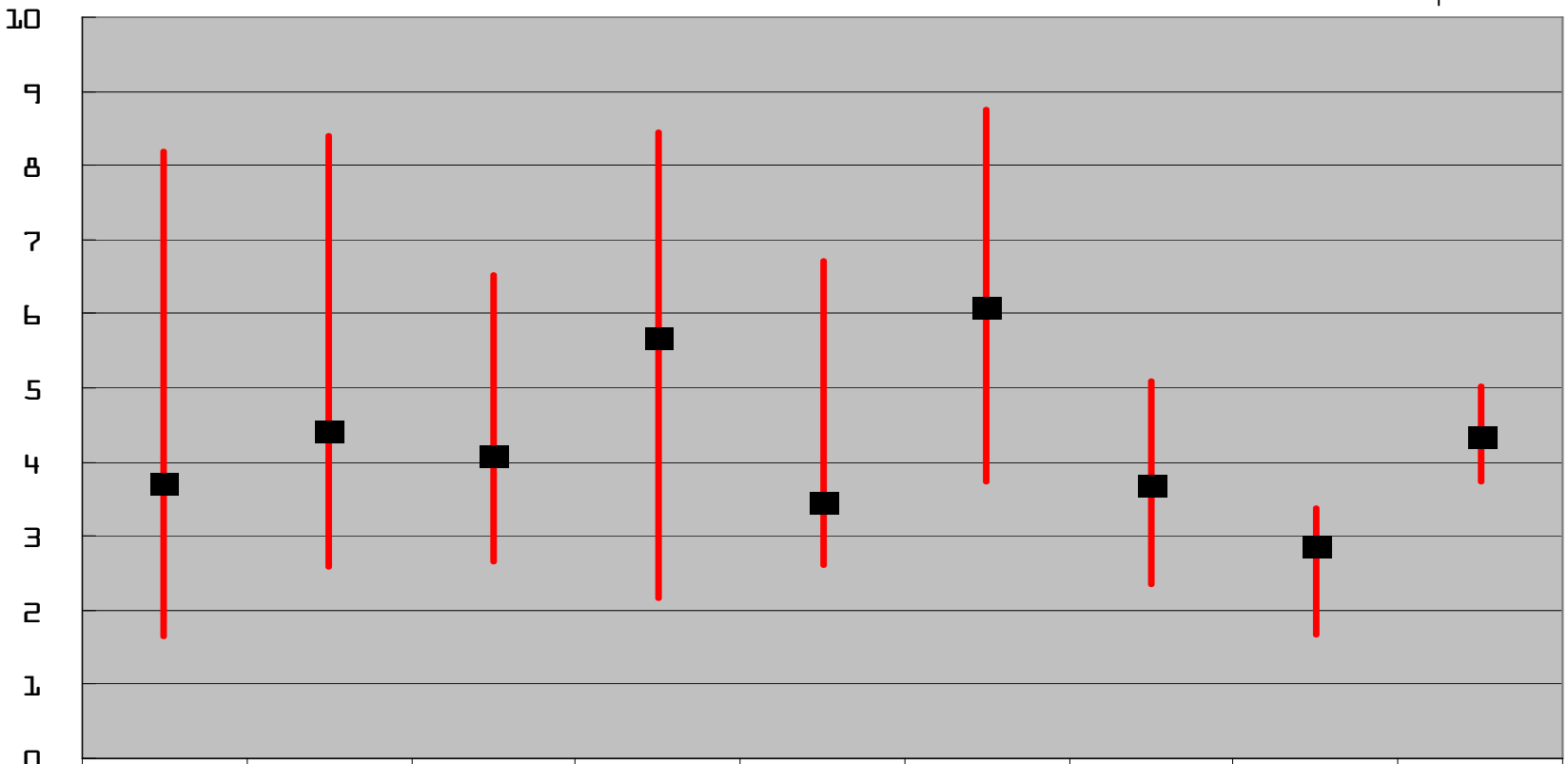


Thermal conductivity ranges of igneous rocks (No. of data: 647)



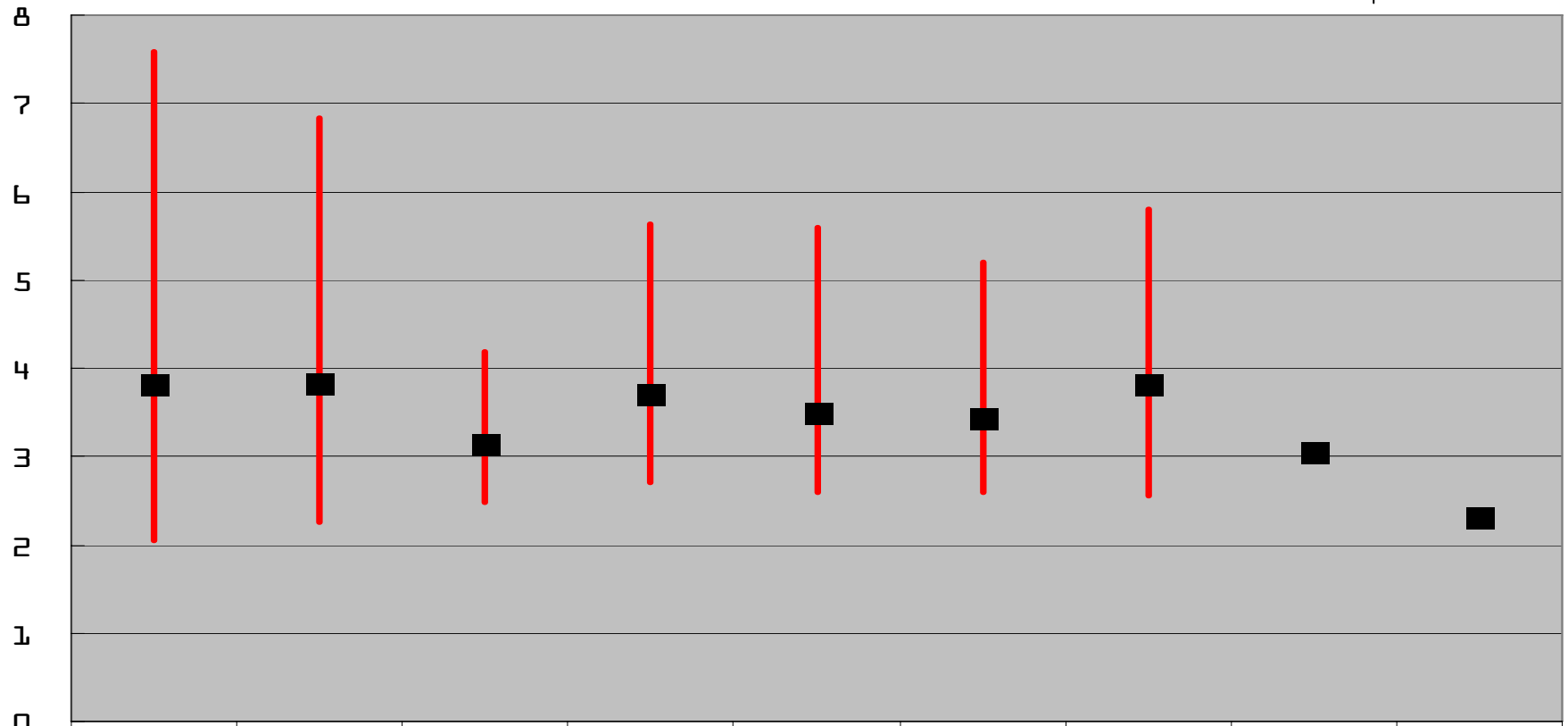
	granite	diorite	feldspar porphyry	quartz porphyry	felsite	granite porphyry	acidic dike	syenite	silicate rock	basic dike rock	felsophyre	felsic dike	gabbro	anorthosite	quartz	halloysite
Minimum	1.82	1.60	2.44	2.80	2.84	2.51	2.04	2.41	3.69	2.73	2.71	3.64	2.50	3.07	7.65	1.72
Maximum	7.71	4.86	4.19	4.45	5.12	5.46	4.91	3.24	4.89	4.12	2.71	3.64	2.50	3.07	7.65	1.72
Mean	3.33	2.87	3.23	3.44	3.41	3.37	3.93	2.76	4.41	3.36	2.71	3.64	2.50	3.07	7.65	1.72
N	512	42	38	13	10	10	6	4	3	3	1	1	1	1	1	1

Thermal conductivity ranges of metamorphic rock (No. of data: 494)



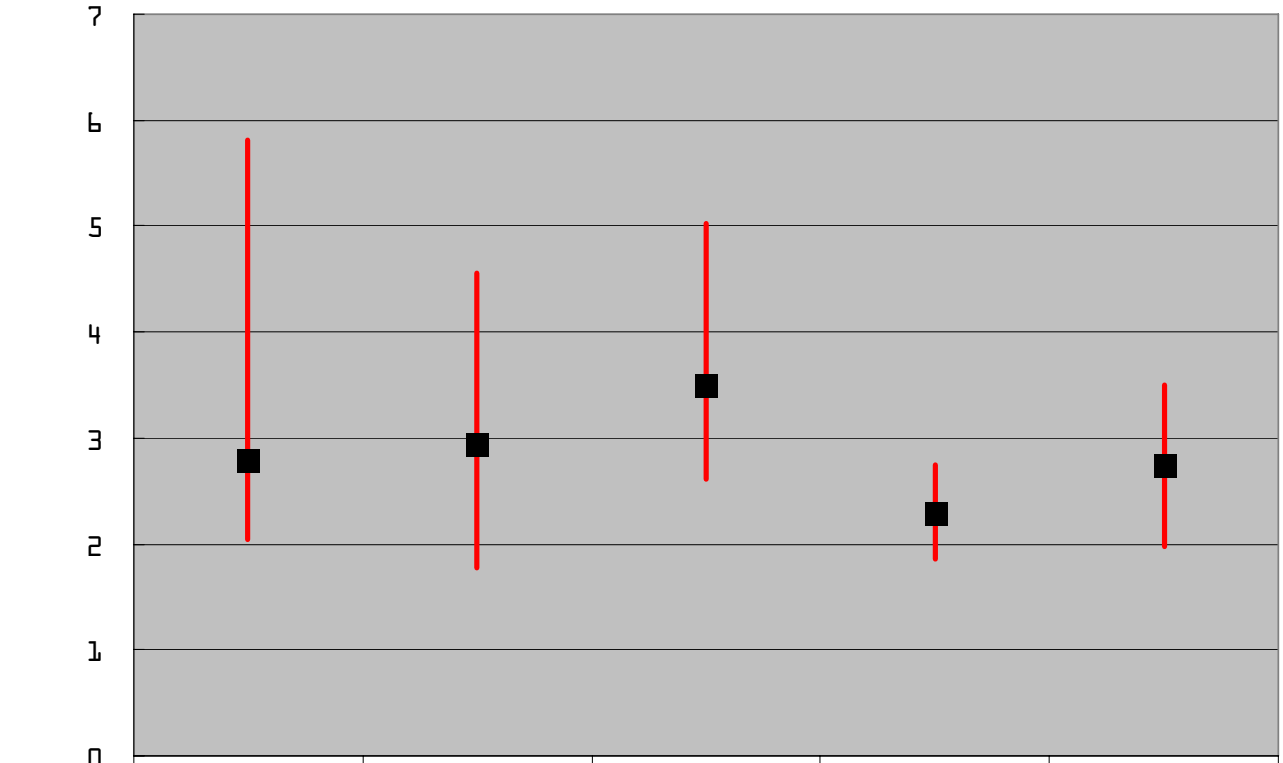
	gneiss	schist	phyllite	quartzite	hornfels	quartz schist	amphibolite	andesitic porphyry	dolomite
Minium	1.64	2.58	2.64	2.17	2.61	3.74	2.34	1.67	3.73
Maximum	8.20	8.39	6.53	8.45	6.71	8.76	5.09	3.39	5.02
Mean	3.72	4.40	4.08	5.69	3.46	6.08	3.68	2.87	4.35
N	359	41	27	25	19	8	6	6	3

Thermal conductivity ranges of sedimentary rocks (No. of data: 291)



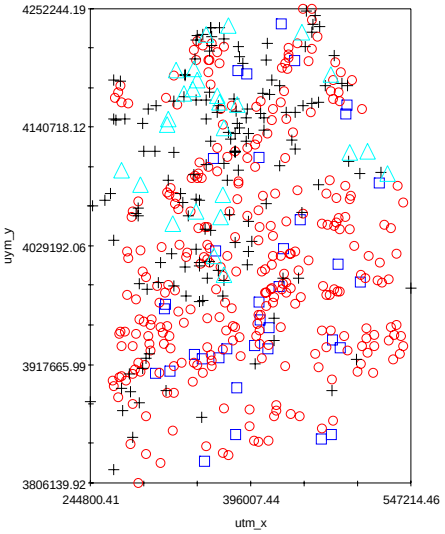
	sandstone	limestone	breccia	mudstone	siltstone	conglomerate	shale	sandic rock	chert
Minimum	2.05	2.25	2.48	2.71	2.59	2.59	2.55	3.04	2.30
Maximum	7.59	6.84	4.18	5.63	5.59	5.21	5.80	3.04	2.30
Mean	3.81	3.83	3.13	3.69	3.49	3.43	3.82	3.04	2.30
N	175	57	17	15	10	8	7	1	1

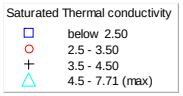
Thermal conductivity ranges of volcanic rocks (No. of data: 128)

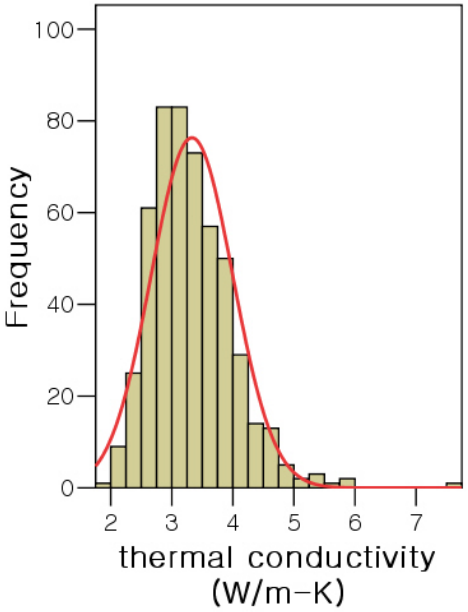


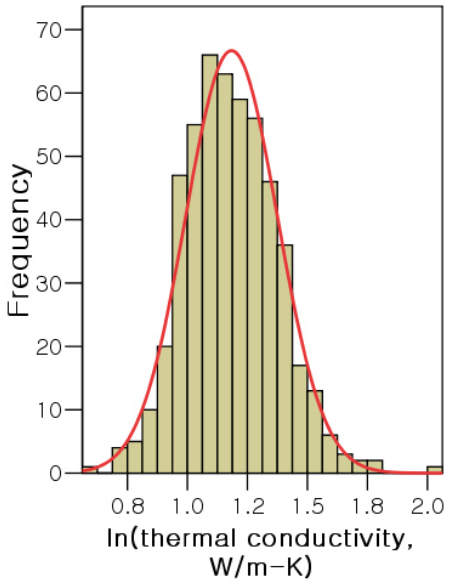
	andesite	tuff	rhyolite	basalt	trachyte
Minimum	2.04	1.77	2.62	1.86	1.98
Maximum	5.81	4.56	5.02	2.75	3.51
Mean	2.80	2.95	3.51	2.29	2.74
N	76	36	10	4	2

General statistical analysis on thermal conductivity of granite







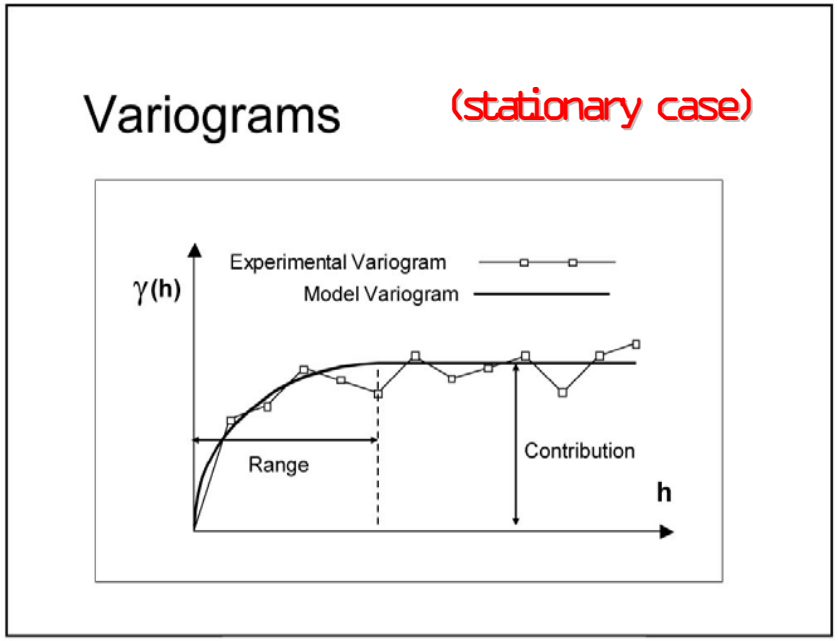


statistics	Non Transformed	Transformed
N	512	512
minimum	1.82	0.60
maximum	5.87	1.77
mean	3.328	1.182
SD	0.668	0.188

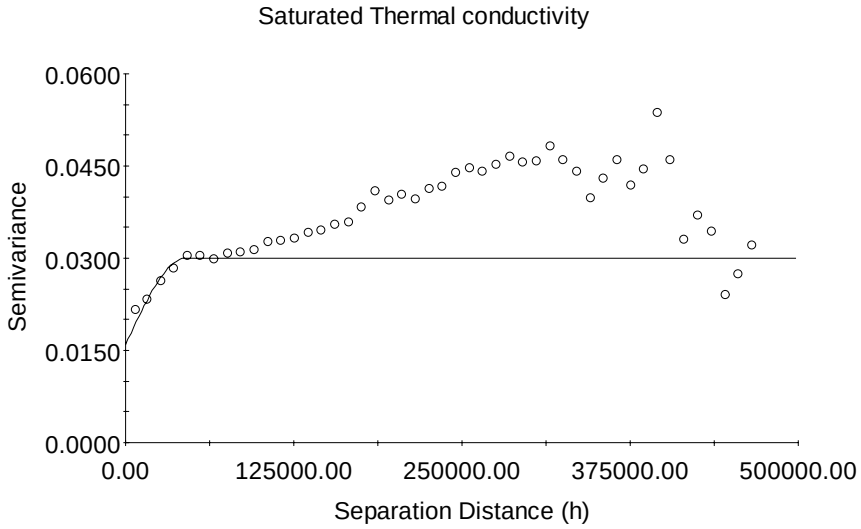
Characterization of spatial patterns: Seimvariogram

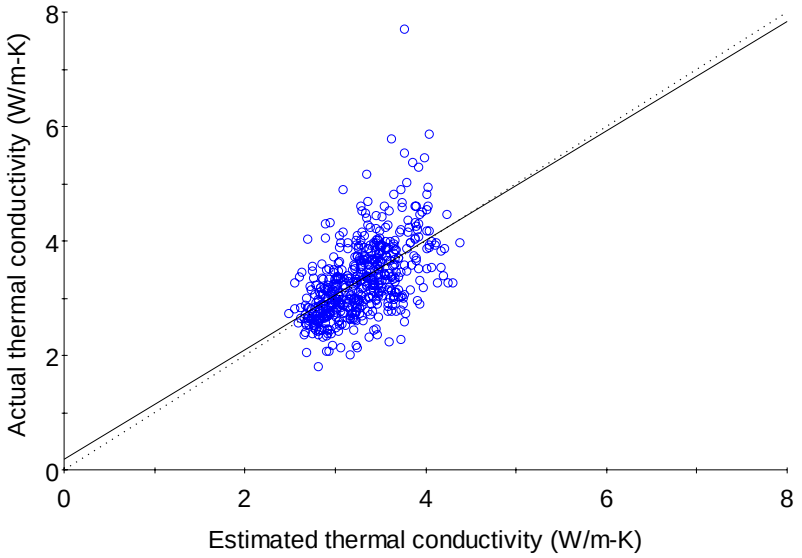
Measure of: average dissimilarity between observations as a function of separation distance + direction

$$\gamma(h) = \frac{1}{2N(h)} \sum_{\alpha=1}^{N(h)} [\rho_m(u_\alpha) - \rho_m(u_\alpha + h)]^2$$



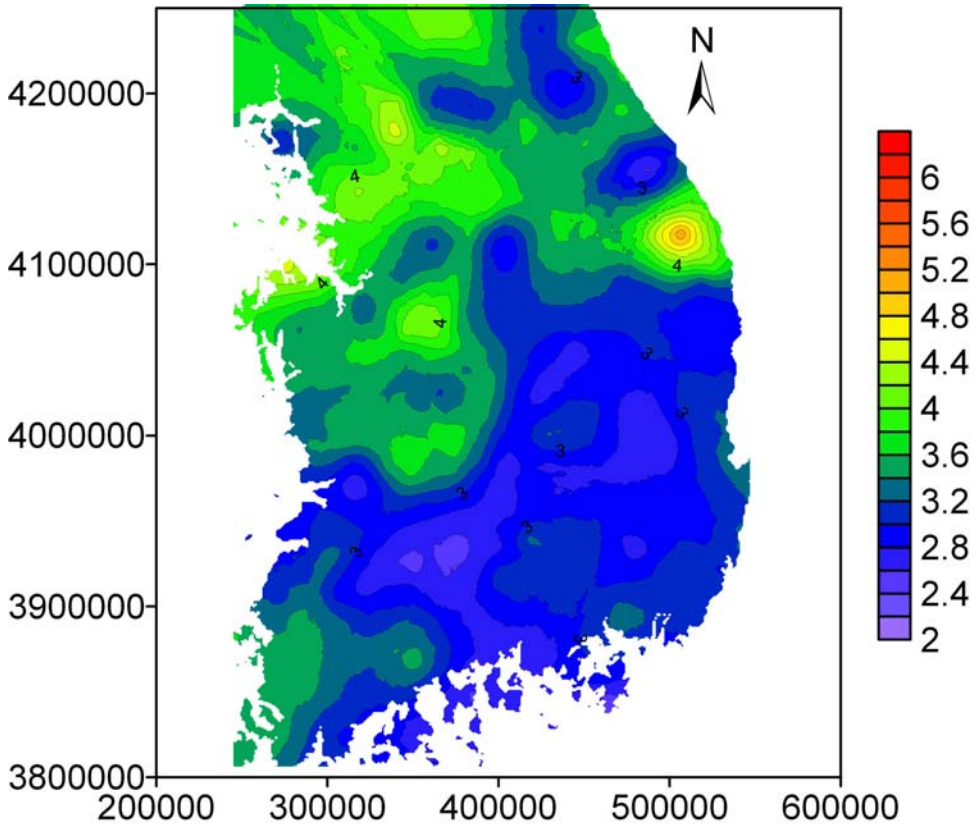
Semivariogram analysis on the saturated thermal conductivity of granite rocks





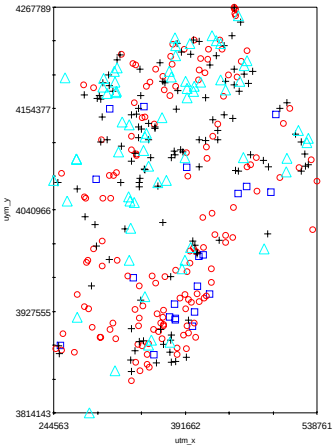
Variogram model type	Nugget variance (Co)	Structural variance Sill (Co+C)	Range (A)
Spherical	0.016	0.030	45000

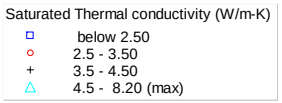
kriging (BLUE) of the thermal conductivity of granite rocks with semivariogram model

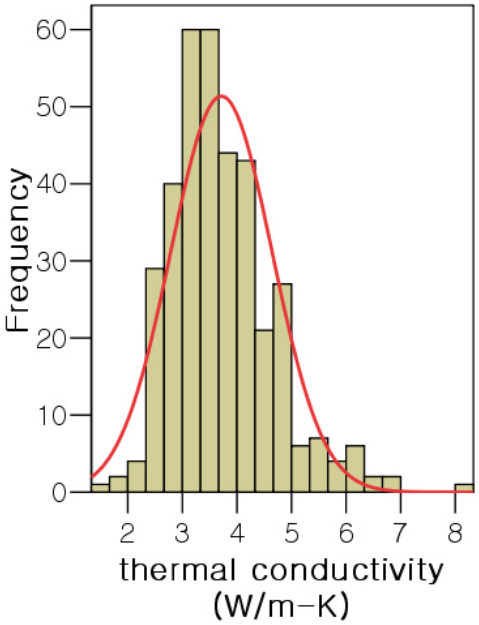


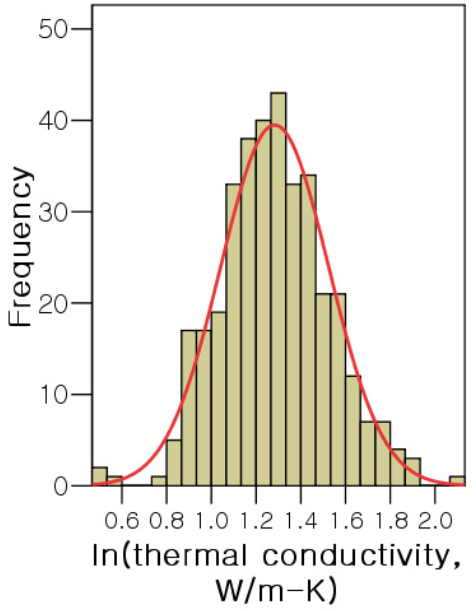
General statistical analysis

Gneiss



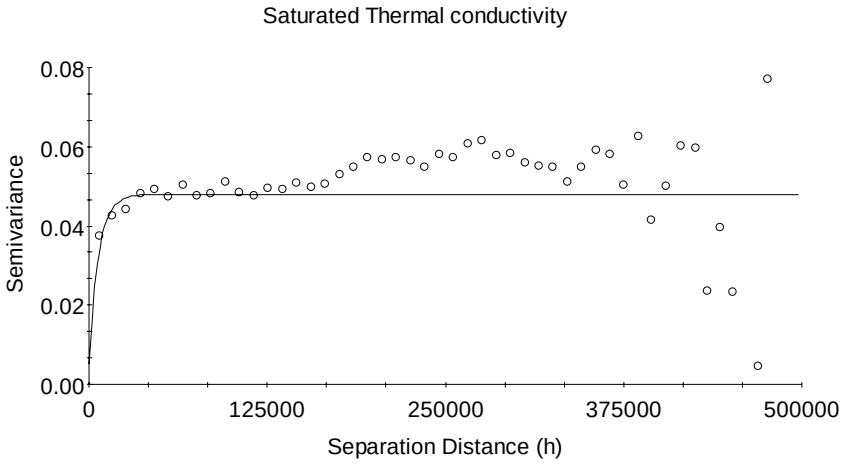




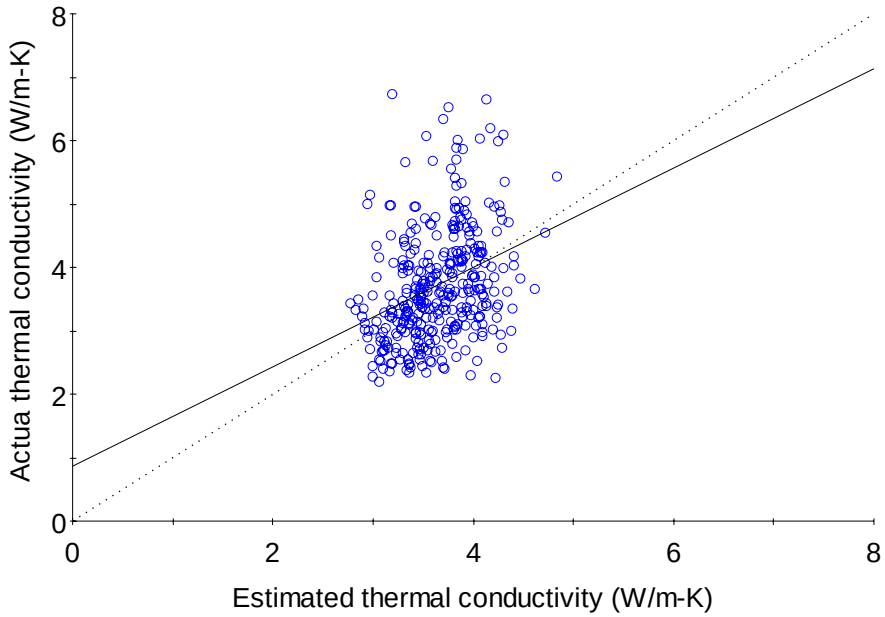


Statistics	Non Transformed	Transformed
N	356	356
minimum	1.64	0.49
maximum	6.75	1.91
mean	3.70	1.280
SD	0.90	0.239

Semivariogram analysis on the saturated thermal conductivity of gneiss

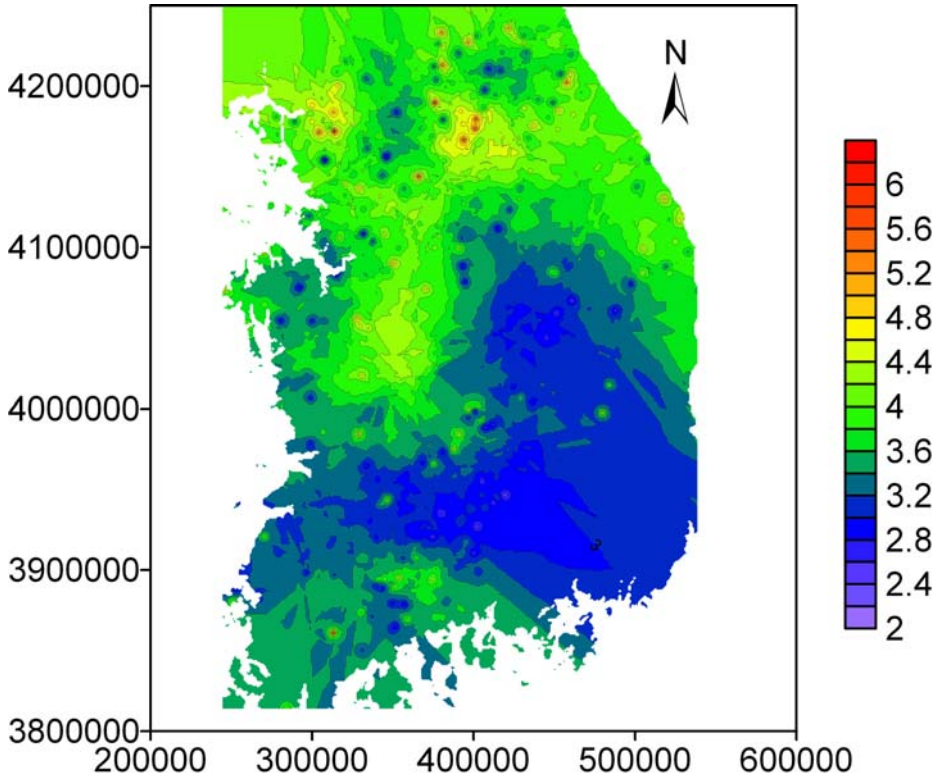


Exponential model (Co = 0.00500; Co + C = 0.04800; Ao = 6666.67; r2 = 0.042; RSS = 6.325E-03)

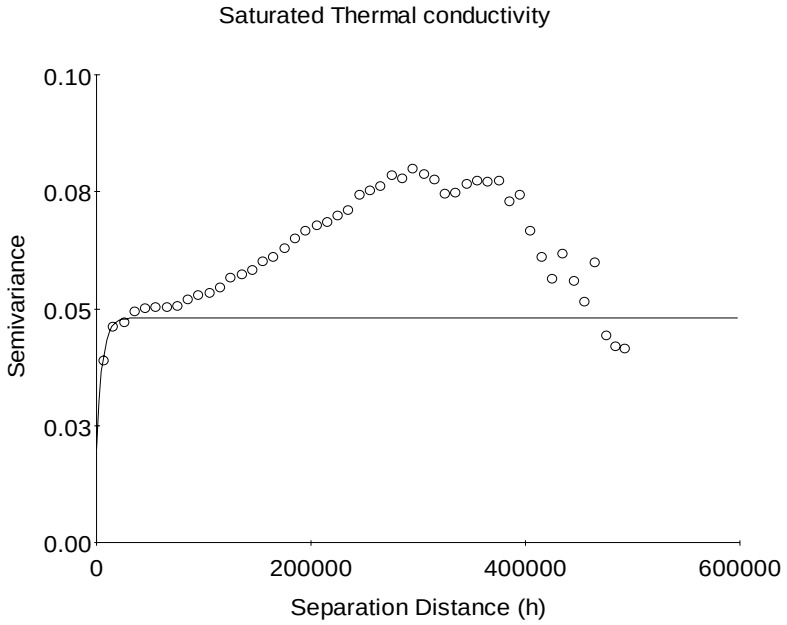


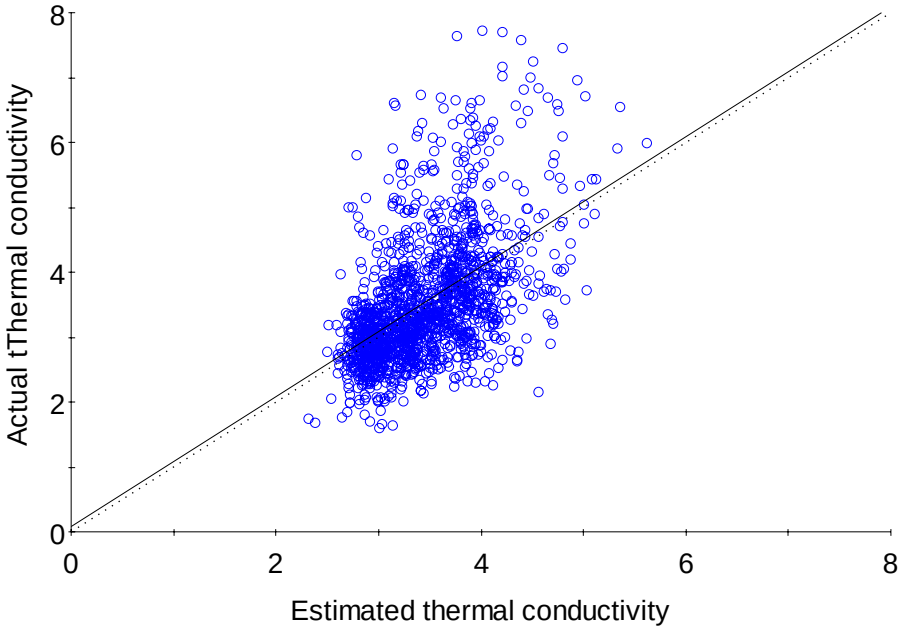
Variogram model type	Nugget variance (Co)	Structural variance Sill (Co+C)	Range (A)
Exponential	0.005	0.048	20000

kriging of the thermal conductivity of gneiss rocks with semivariogram model



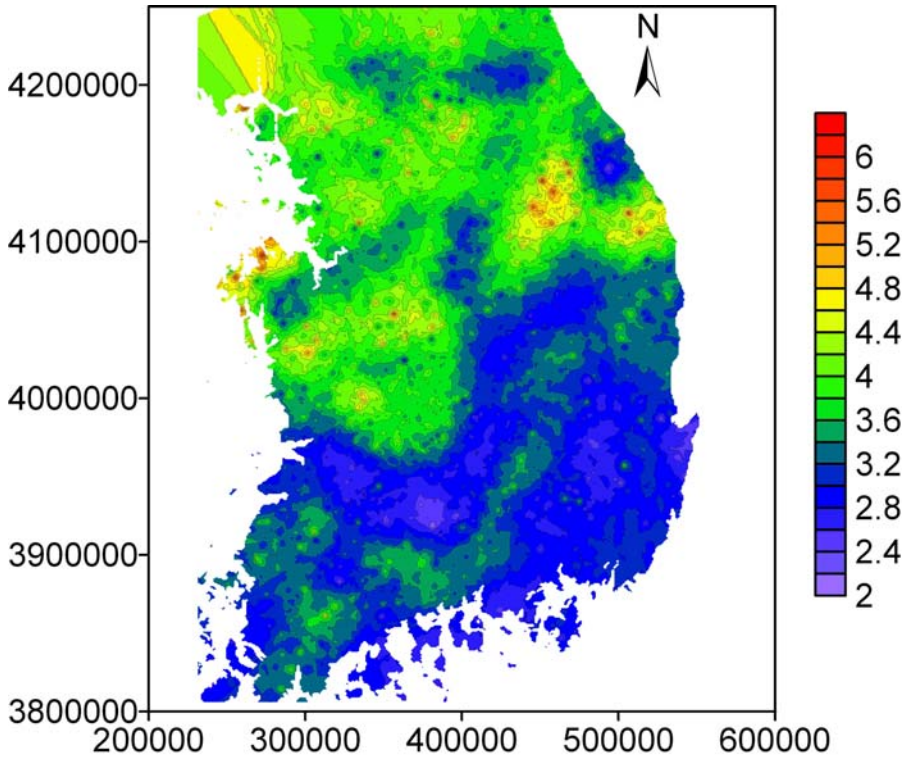
Semivariogram analysis on the saturated thermal conductivity of total sampled rocks





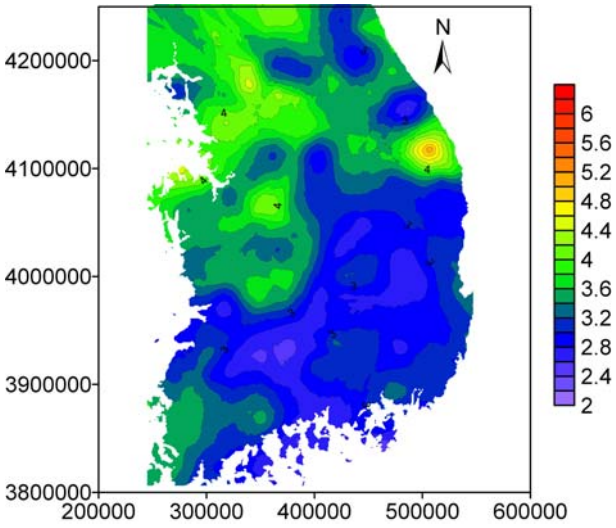
Variogram model type	Nugget variance (Co)	Structural variance Sill (Co+C)	Range (A)
Exponential	0.02	0.048	16000

Thermal conductivity distribution map of total rocks by kriging

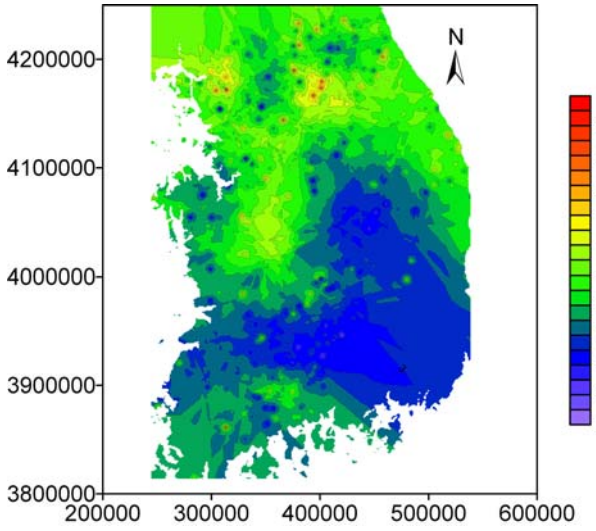


Thermal conductivity distribution maps

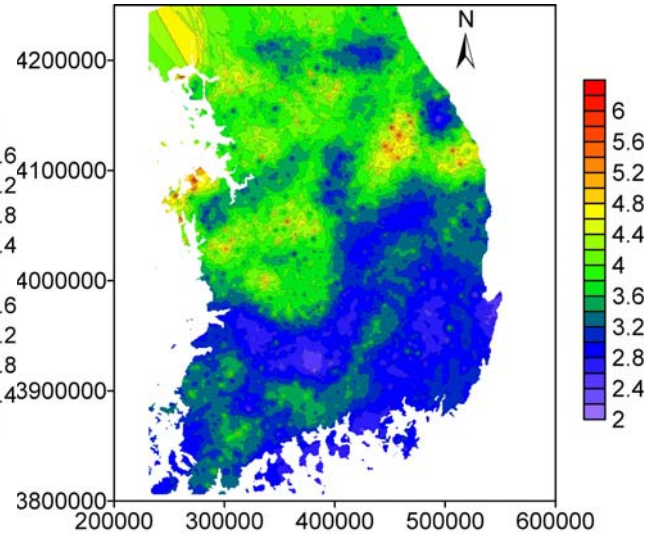
Granite



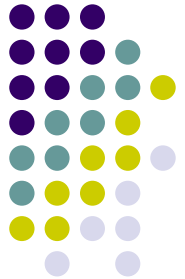
Gneiss



Total



Statistical equations for cross validation tests



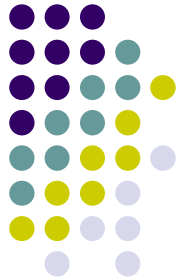
$$RE = \frac{Z(x) - Z^*(x)}{\sqrt{\sigma_k^2}} \qquad MRE = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{Z(x) - Z^*(x)}{\sqrt{\sigma_k^2}}\right]_i \approx 0$$

$$VRE = \frac{1}{N-1}\sum_{i=1}^{N}(RE - MRE)_i^2 \qquad SDRE = \sqrt{VRE} \approx 1$$

$$SRMSE = \sqrt{MSE} \qquad MKV = \frac{1}{N}\sum_{i=1}^{N}(\sigma_k^2)_i$$

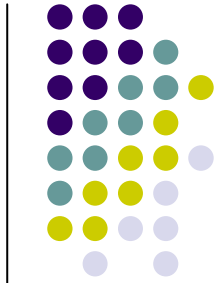
$$VE = \frac{1}{N-1}\sum_{i=1}^{N}(Error - ME)^2 = MKV$$

Cross validation test results



statistics	granite	gneiss	total rock
ME	0.0001	0.0000	0.0001
MSE	0.5739	0.0218	10.9838
SRMSE	0.1498	0.2848	0.2561
MKV	0.3153	0.7789	0.6783
MRE	0.0224	0.0811	0.0656
VE	0.3153	0.7811	0.6787
VRE	0.9991	0.9962	1.0130
SDRE	0.9995	0.9981	1.0065

Conclusions



- Histograms of the thermal conductivity of granite, gneiss and total rocks show lognormal distributions.
- Most of the porosity of rock specimens are less than 10%, and shows weak relationship with thermal conductivities.
- The semivariograms of the thermal conductivities of granite, gneiss and total rocks shows weak stationarity.
- In cross validation tests of estimated thermal conductivities, each scatter plot and statistics show similar agreements between actual and estimated values.