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Joint inversion of heat flow, elevation, geoid height:

Determination of deep temperature field in geothermal areas.

Alexandrino, C.H. and Hamza,V. M.

Observatório Nacional / MCT
Rio de Janeiro, Brazil

Scope of the present work

An understanding of the deep temperature field of the crust is often considered important in exploration and assessment of geothermal resources.

Usually results of regional heat flow studies are employed for this purpose. However, this approach does not always lead to satisfactory results because of problems arising from limited availability of suitable boreholes.

In the present work we propose a method for extracting supplementary information on deep temperature field, from a knowledge of the conditions of thermal isostasy. In practice, this amounts to joint inversion of data sets on heat flow, elevation and geoid height. The advantage here is that suitable high density data on elevation and geoid height can be acquired with relative ease, and in addition, these do not depend on availability of boreholes.

Joint Inversion of Heat flow, Elevation and Geoid Height

The assumptions are:

1- Conditions of thermal isostasy prevail;

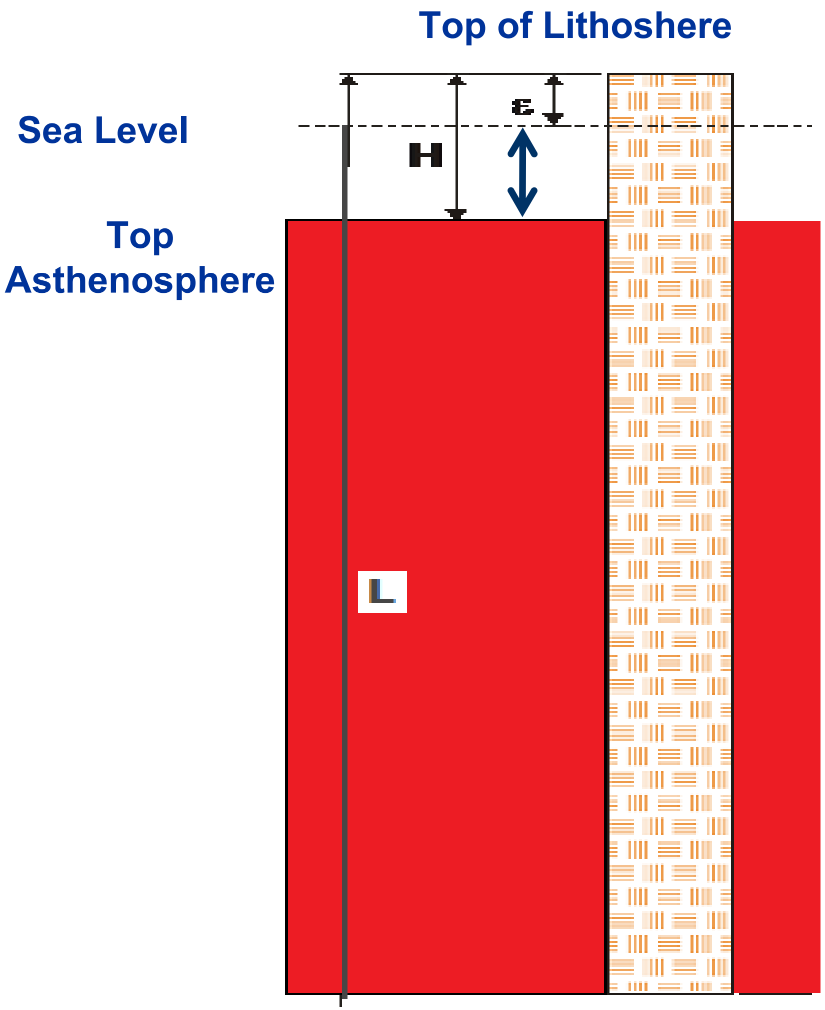
2- Lateral changes in density are small compared to vertical changes.

Under such conditions the geoid height is proportional to the dipole moment of the vertical distribution of density, which in turn is also temperature dependant.

(Ockendon e Turcotte, 1977; Turcotte e Oxburgh, 1982):

$$N = -\frac{2\pi G}{g} \int_{cL} z \, . \, \rho(z) \, dz + N_0$$

Mass balance relations for lithosphere under conditions of thermal isostasy (Lachenbruch and Morgan, 1990)



$$L\rho_l = (L - H)\rho_a \qquad \rho_a H = (\rho_a - \rho_l)L \qquad (1)$$

$$\varepsilon = H - H_0 \qquad \varepsilon > 0 \qquad (2)$$

$$\varepsilon = \frac{\rho_a - \rho_l}{\rho_a} L - H_0, \qquad \varepsilon > 0 \qquad (3)$$

L – thickness of lithosphere (km)
ρ_l – density of lithosphere (g/cm^3)
ρ_a – density of asthenosphere (g/cm^3)
H – difference in height (km)
H_0 – Elevation of asthenosphere (km)
e – Elevation above sea level (km)

Isostatic balance – Formal relations for regions affected by tectonic processes

Geope of ‘dry’ asthenosphere

$$\varepsilon + H_0 = \frac{\varepsilon_0 + H_0}{\beta} + \left(1 - \frac{1}{\beta}\right)\Sigma, \qquad \varepsilon > 0$$

Hydrogeope of asthenosphere

$$\varepsilon + \Delta_0 = \frac{\varepsilon_0 + \Delta_0}{\beta} + \left(1 - \frac{1}{\beta}\right)\frac{\rho_a}{\rho_a - \rho_w}\Sigma, \qquad \varepsilon < 0$$

where β is the stretching factor, Σ is the contributions by other processses

Consider a lithosphere composed of four layers:

Sea water, Crust, Lithospheric Mantle and Asthenosphere.

The relations between elevation and crustal thickness are:

$$z_c = \frac{\rho_a L_0 - E\rho_c + z_L(\rho_m - \rho_a)}{(\rho_m - \rho_c)}$$

for E > 0

$$z_c = \frac{\rho_a L_0 + E(\rho_c - \rho_w) + z_L(\rho_m - \rho_a)}{(\rho_m - \rho_c)}$$

for E < 0

The temperature at the base of the crust may be expressed using computed values of depth to base of lithosphere (z_L)and basal heat flow (as was done by Fullea et al, 2007) or measured values of surface heat flow (as proposed by Alexandrino and Hamza, 2008).

The main advantage of this latter approach is that it provides more reliable information on temperature field and on thermal properties at depth in the lower crust.

For Oceanic regions (Alexandrino & Hamza, 2008)

$$\left(\frac{1}{2}\rho a\,\alpha\,\theta-\frac{1}{2}\rho a\,\alpha\,Ta\,kc\right)zL^2+\Big(zc\,\rho a\,kc-zc\,\rho c\,kc$$
$$-Lo\,\rho a\,kc+\frac{1}{2}\rho a\,\alpha\,\delta+\frac{1}{2}zc\,\rho a\,\alpha\,Ta\,kc+E\,\rho c\,kc$$
$$-zc\,\rho a\,\alpha\,\theta-\frac{1}{2}\rho a\,\alpha\,Ta\,zc\,\Delta k-\frac{1}{2}\rho a\,\alpha\,Ta\,E\,km$$
$$+E\,\rho w\,kc\Big)zL-zc^2\rho c\,\Delta k+zc^2\rho a\,\Delta k+E^2\rho c\,km$$
$$+E^2\rho w\,km+\frac{1}{2}zc^2\rho a\,\alpha\,Ta\,\Delta k-Lo\,\rho a\,E\,km+zc\,\rho a\,E\,km$$
$$+\frac{1}{2}zc^2\rho a\,\alpha\,\theta-\frac{1}{2}zc\,\rho a\,\alpha\,\delta-zc\,\rho c\,E\,km-Lo\,\rho a\,zc\,\Delta k$$
$$+E\,\rho w\,zc\,\Delta k+E\,\rho c\,zc\,\Delta k+\frac{1}{2}zc\,\rho a\,\alpha\,Ta\,E\,km=0$$

Simplified as (Fullea et al, 2007):

$$z_L^2(T_a k_c-\theta)+z_L\left(z_c\big(T_a(k_m-2k_c)+2\theta\big)-\delta+T_a E k_m-\frac{2k_c}{\rho_a\alpha}\left[(\rho_a-\rho_c)z_c+\eta\right]\right)$$
$$+\left(z_c\left[\delta-T_a\left(z_c\Delta k+Ek_m\right)-z_c\theta\right]-\frac{2}{\rho_a\alpha}\left[\left(z_c\Delta k+Ek_m\right)\left(\eta+\left(\rho_a-\rho_c\right)z_c\right)\right]\right)=0$$

The relation for the thickness of the lithosphere is:

$$z_L^2\left(T_a k_c-\theta\right)+z_L\left(z_c\left(T_a\left(k_m-2k_c\right)+2\theta\right)-\delta+T_a E k_m-\frac{2k_c}{\rho_a\alpha}\left[\left(\rho_a-\rho_c\right)z_c+\eta\right]\right)$$
$$+\left(z_c\left[\delta-T_a\left(z_c\Delta k+Ek_m\right)-z_c\theta\right]-\frac{2}{\rho_a\alpha}\left[\left(z_c\Delta k+Ek_m\right)\left(\eta+\left(\rho_a-\rho_c\right)z_c\right)\right]\right)=0$$

The relation for geoid height becomes:

$$N=-\frac{\pi G}{g}\left[\begin{array}{l}\rho_w E^2+\dfrac{2\beta}{3}\left(z_c^3-E^3\right)+\left(\beta E+\rho_c^T\right)\left(z_c^2-E^2\right)\\ +\left(z_{\max}^2-z_c^2\right)\rho_a+\rho_a\alpha\dfrac{T_a-T_{mh}}{3}\left[\left(z_L-z_c\right)\left(z_L+2z_c\right)\right]\end{array}\right]+N_0$$

The combined solution of these equations alllow analysis of elevation and geoid height under conditions of thermal isostasy

Iterative schemes are necessary because of the non-linearity of the equations

Computational steps:

1- Estimate the initial values for z_c and $z_{L,}$ assuming constant density for crust and mantle;

2- Use the initial value of z_c for calculating the depth to the base of the lithosphere, which couples isostasy to the thermal field;

3- Calculate the temperature at the base of the crust (T_{mh}) using values of z_c and z_L of step 2;

4- Calculate the the geoid height using z_c, z_L and T_{mh} obtained in step 3;

5- Determine the residual anomaly (calculated – observed);

6- Change the value of z_c and repeat the process until the resiual anomaly is minimized.

1 - Modules dof Input Data

Input Parameters		
Density at top	ρc t	2640,00
Density at bottom	ρc b	2920,00
Average density	ρc m	2780,00
Mantle density	ρm	3293,92
Density asthenosphere	ρa	3200,00
Density of water	ρw	1030,00
Compensation Level	z max	300000,00
Coeficient of expansion	α	3,50E-05
Radiogenic heat	Hs	8,20E-07
D parameter	hr	1,05E+04
Crustal conductivity	kc	2,5000
Mantle Conductivity	km	3,2000
Surface temperature	Ts	20,00
Temp. base lithosphere	Ta	1350,00
Elevation	*E*	500,00
Geoide Astenospheric	Lo	2320,00
Gravitational Constant	G	6,67E-11
PI	pi	3,14
acceleration	g	9,79
Radiogenic heat	f	83,79

Thickness Lithosphere	
Equation 12	
eta	-8,299E+06
	3,241E+03
a	3,241E+03
Term 1	-1,769E+08
Term 2	2,160E+06
Term 3	1,349E+08
	-3,096E+08
b	-3,096E+08
Term 4	2,355E+12
	1,104E+12
c	1,250E+12
	1,250E+12
delta	2,822E+08
r1	4,2252E+03
r2	9,1304E+04
Z_L	**9,130E+04**
Zc	**2,695E+04**

Temperature Moho	
Equation 10	
θ	133,79
delta	1,1860E+08
deltaK	0,70
T_{Moho}	**511,45**

Hydro Geope Anomaly	
Equation 13	
Beta	1,020E-02
a	-2,142E-11
b	2,575E+08
c	1,331E+11
d	1,921E+12
e	2,857E+14
g1	3,131E+01
g2	9,344E+09
g	2,925E+11
Sum	2,880E+14
product	-6,1707E+03
N =	**-6,1707E+03**

Reference Hydro Geope	
Equation A4 (case b)	
π G / g	2,142E-11
((ρm-ρw)/(ρm-ρa)) E	1,205E+04
2 ρa L_0	1,485E+07
(ρa-ρw) E	2,115E+06
2 ρa L_0 + (ρa-ρw) E	1,696E+07
Product 1	2,044E+11
Z_0^2 ρa	2,880E+14
(ρa L_0)^2 / (ρm-ρa)	5,869E+11
Sum	2,888E+14
Noc =	6,1872E+03

Equation A1	
Termo 1	257500000,00
Termo 2	1,8351E+12
Termo 3	2,5284E+13
Termo 4	2,6132E+14
Soma	2,8844E+14

Equation A2	
a	7,424E+06
b	8,050E+05
c	8,5751E+06
soma	1,680E+07
d	6,539E+02
divisão	2,570E+04
Zc	**2,570E+04**

Equation A3	
kapa	8,299E+06
Termo 1	2,381E-03
Termo 2	5,472E+00
Termo 3	6,8873E+13
Termo 4	-1,863E+14
Termo 5	2,5517E+14
Z_L	**1,0873E+05**

Equation A4 (case a)	
a	2,142E-11
b	-4,375E+08
c	2,880E+14
d	1,640E+11
Soma	2,878E+14
No - N =	6166,73

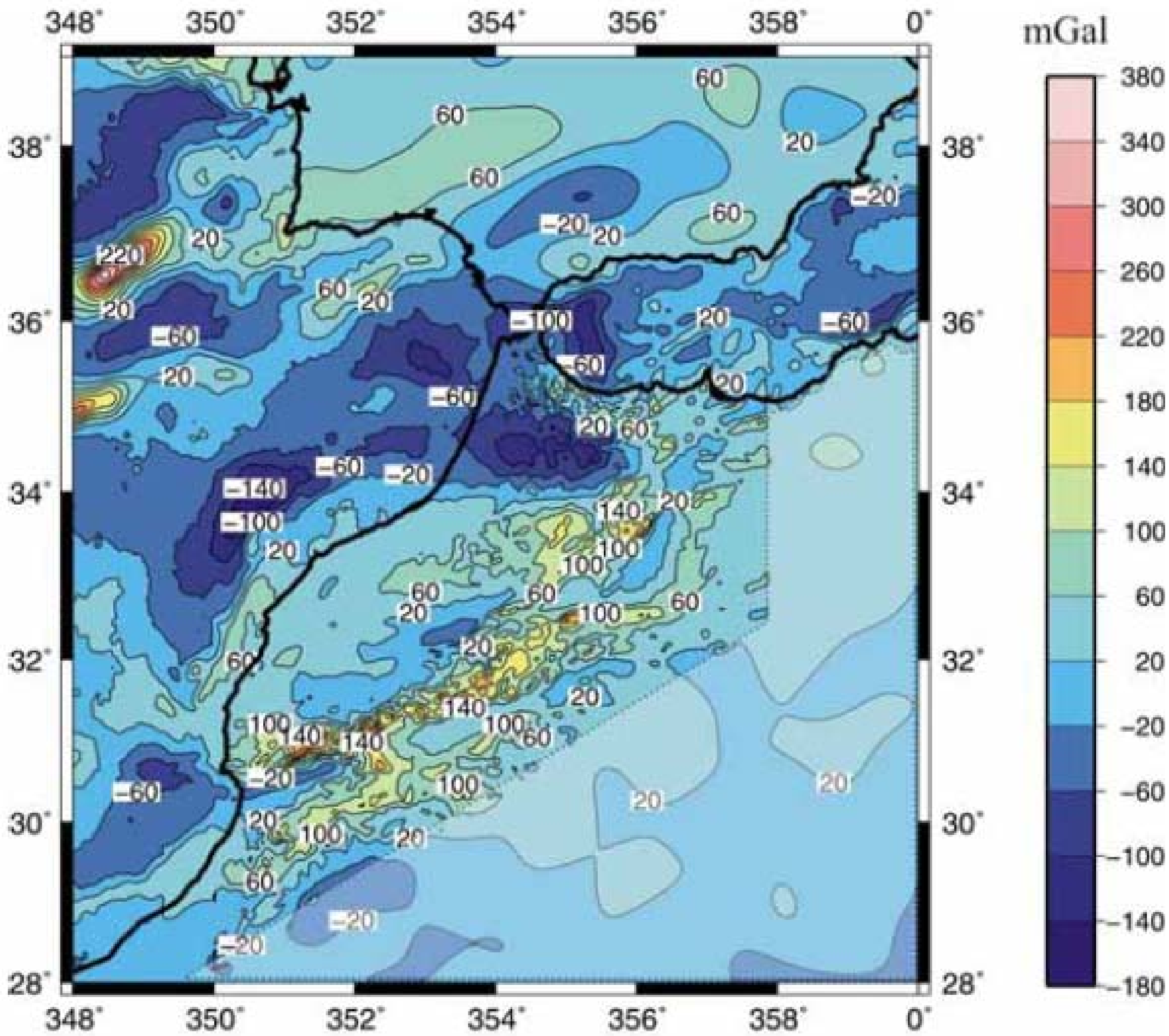
2 – Modules for Iterative Steps

Estimates of Iterative Process		
Moho depth (km)	zc ref	26,95
Base of lithosphere (km)	zL ref	91,30

Hydro Geope Residual	
Geoide Height - calculated	-4,00
Geoide Height - observed	-4,00
Residual (observed - Calculated)	0,00

Mantle density	
Equation 11	
ρm m	3246,96

Study area of Fullea et al 2007
(include segments of North Africa and of the Mediterranean)



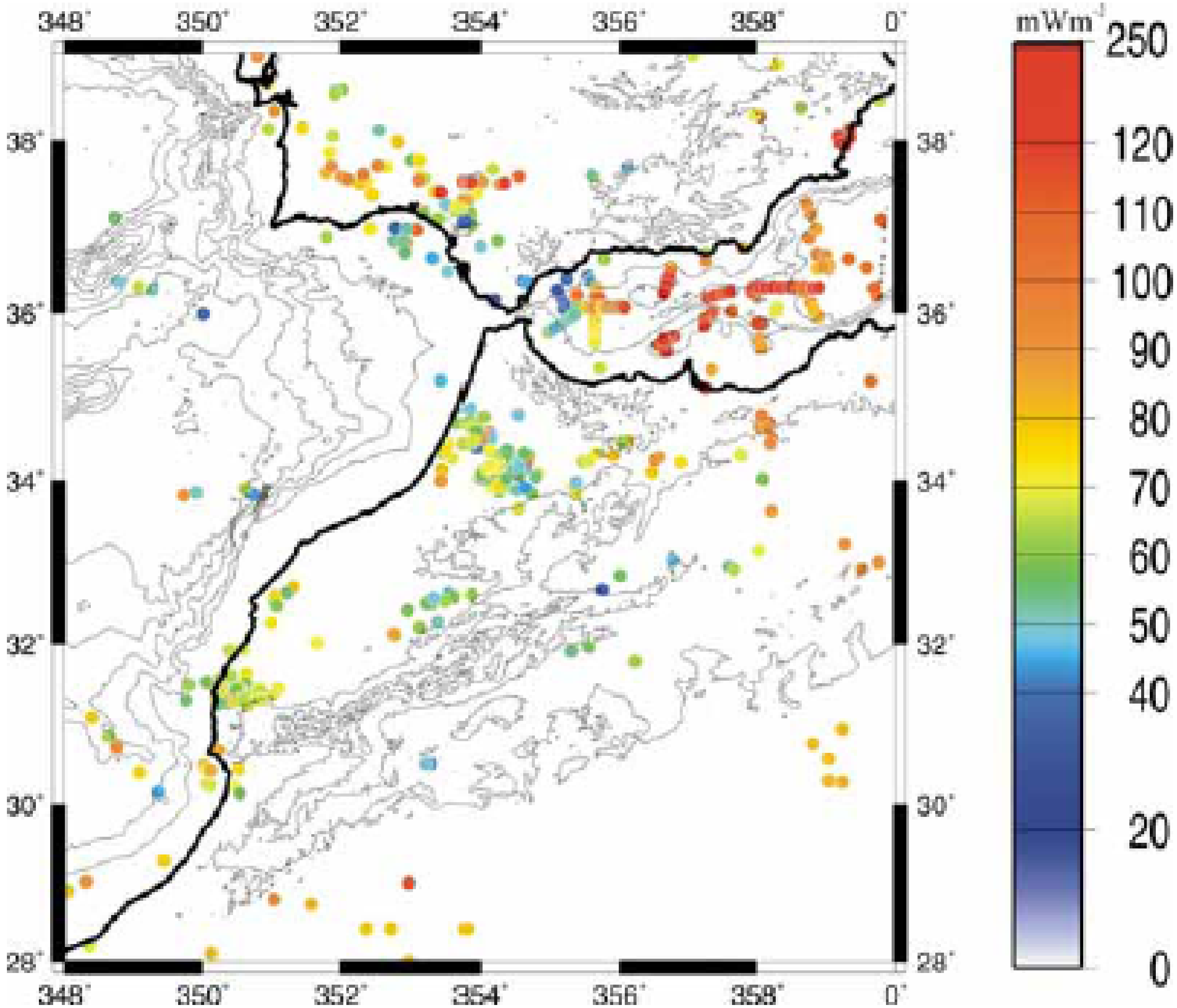
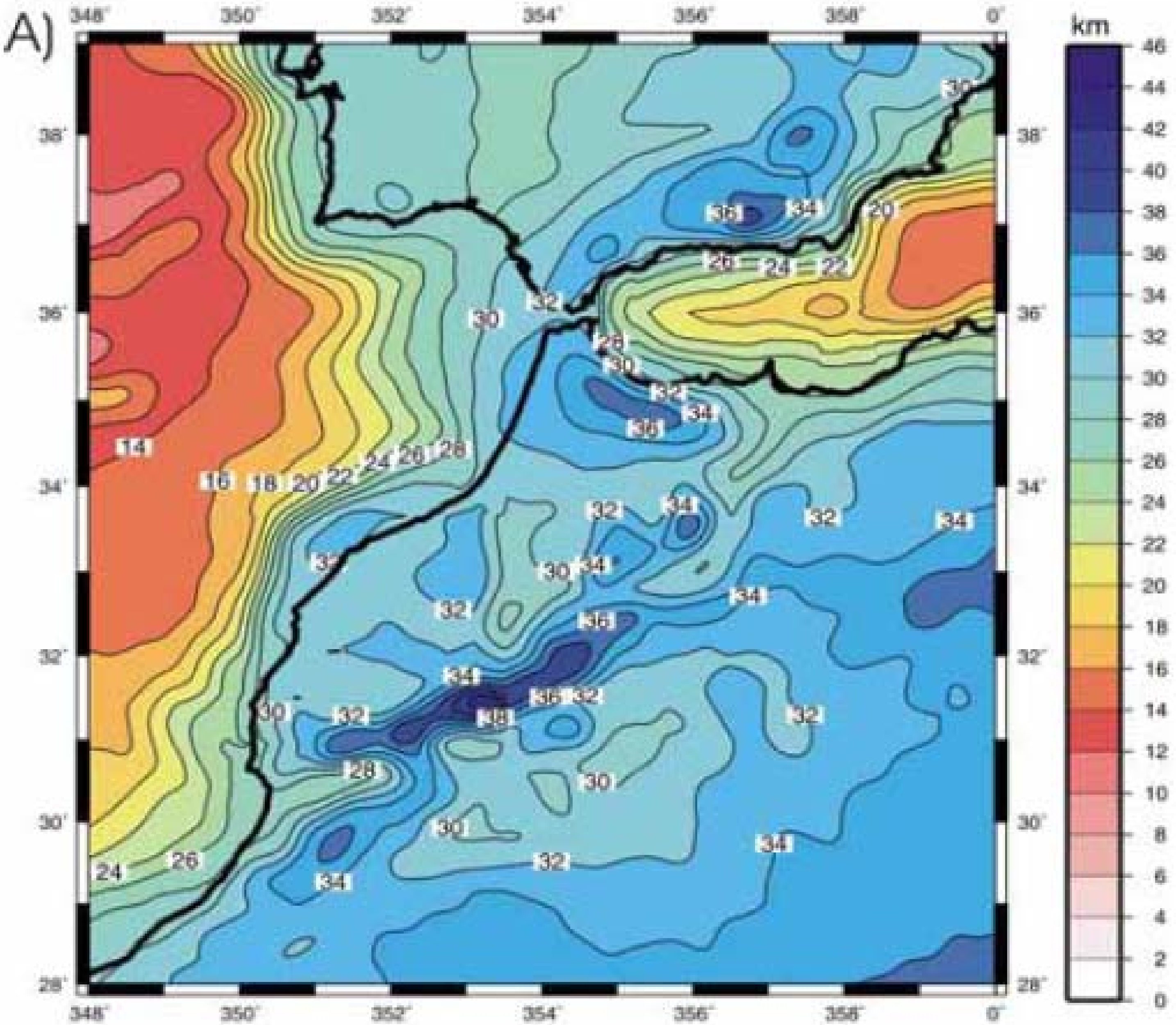
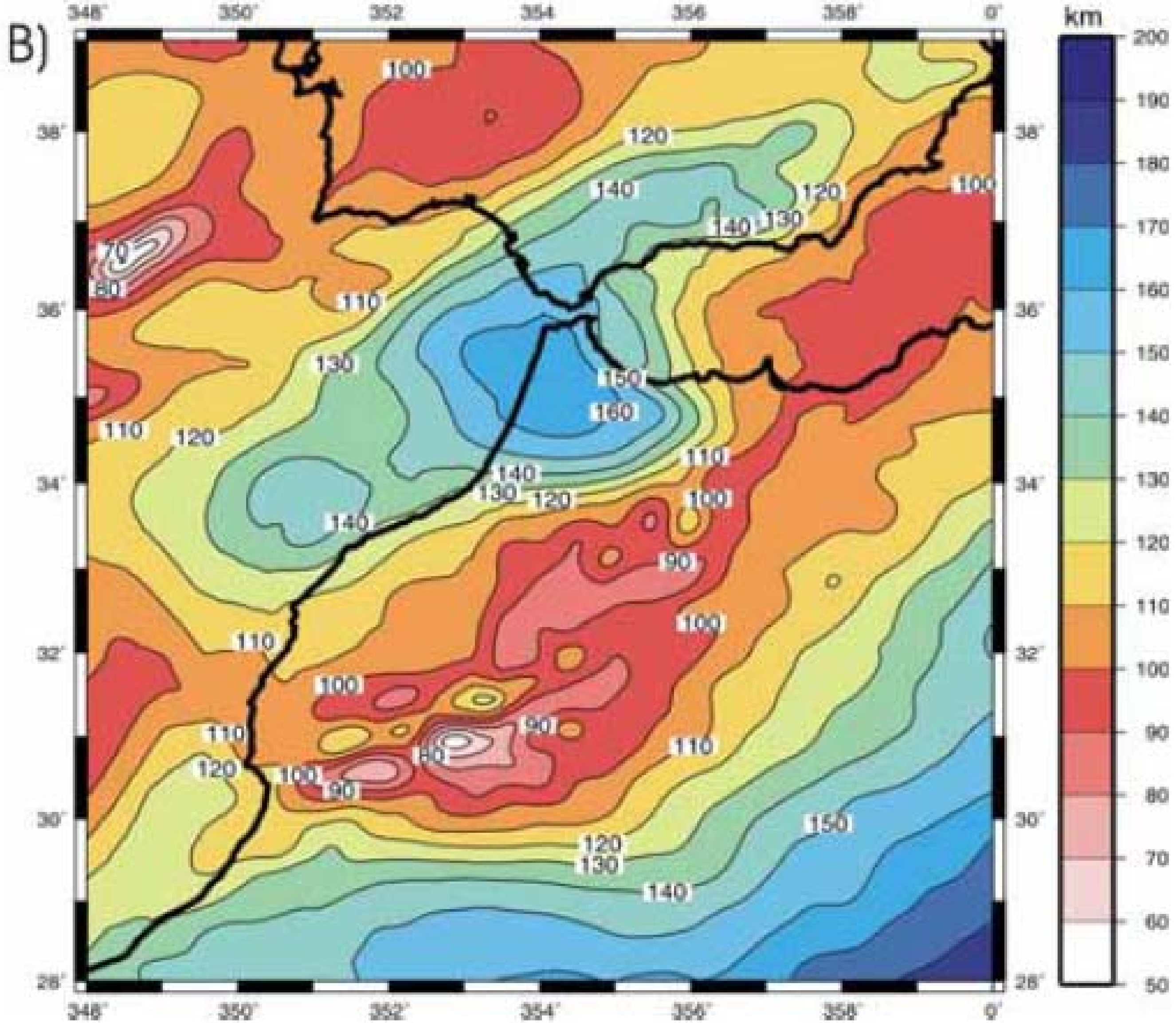


Fig. 3.14 *Surface heat flow data of the Gibraltar Arc System region. Data from Fernàndez et al., (1998), Polyak et al., (1996), Verzhbitsky and Zolotarev (1989), and Rimi et al., (1998).*

Thickness of crust derived from elevation and geoide height



Thickness of lithosphere derived from elevation and geoid height (This is related to the deep temperature field of the crust)



The limitations in the approach of Fullea et al (2007)

1- Does not use surface heat flux as an input parameter;

2- The model results are rather insensitive to changes in the thermal field;

3- Does not admit vertical variations in thermal properties;

4- Does not provide information on thermal field at depth

Improvements proposed in the present work (Alexandrino and Hamza, 2008)

1- Surface heat flow is considered as an input parameter;

2- Allows vertical variations in thermal properties of the crust;

3- Provides information on thermal field at depth;

4- Employs a multiple iteration process for determining crustal temperatures, based on simultaneous fit to elevation, geoid height, and heat flow.

1 – Input data Modules

Fullea et al, 2007

Moho Temperature	
Equation 10	
θ	133,79
delta	1,1860E+08
deltaK	0,70
T_{Moho}	**511,45**

Alexandrino & Hamza, 2008

Moho Temperature	
Equation 10b	
θ	139.32
delta	1.38E+08
deltaK	1.09
Param B	6.82E-04
Param C	6.32E-10
Heat $flow_{Surface}$	5.00E-02
Moho Heat Flux	4.21E-02
Conductivity	3.00
T_{Moho}	**451.32**

2 – Modules of Iterative Processes

Fullea et al, 2007

Single iteration - Initial Estimates		
Moho Depth (km)	**zc ref**	**26,95**
Lithosphere Thickness (km)	**zL ref**	**91,30**

Alexandrino & Hamza, 2008

Multiple Iteration Process		
Moho Depth (km)	**zc ref**	**26.23**
Lithosphere Thickness (km)	**zL ref**	**83.80**
Feedback of Z_L based on heat flow		**83.88**

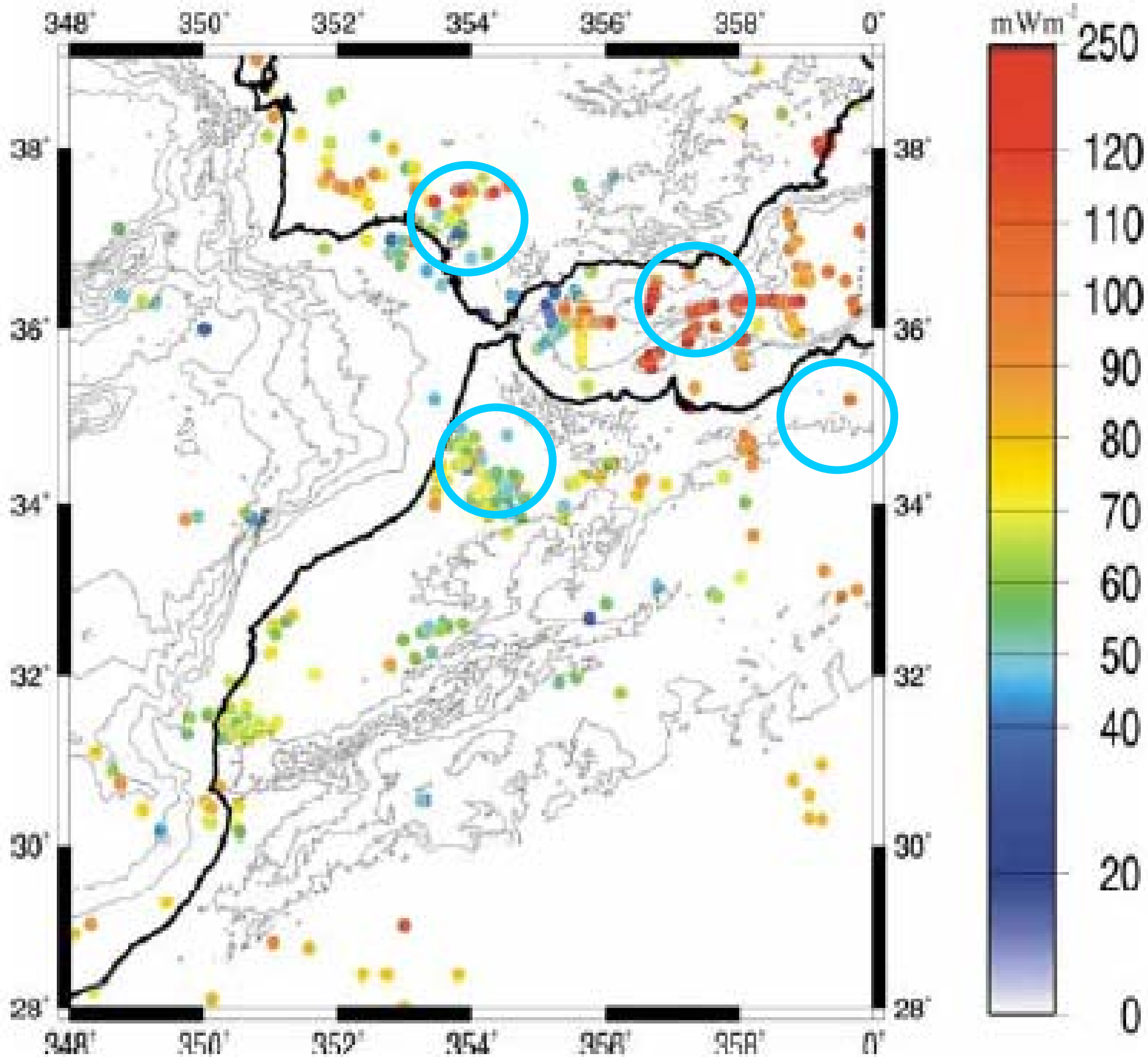
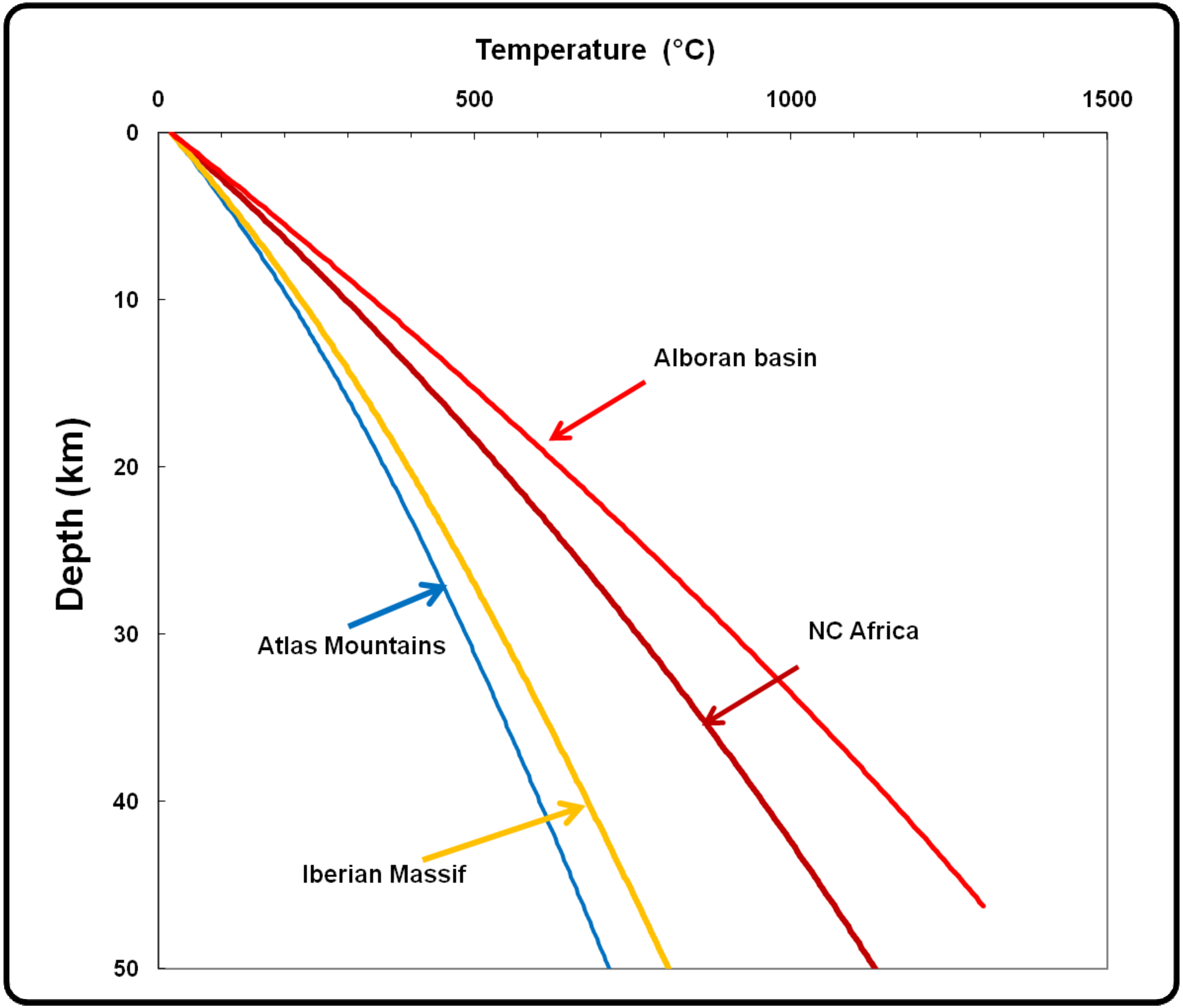


Fig. 3.14 *Surface heat flow data of the Gibraltar Arc System region. Data from Fernàndez et al., (1998), Polyak et al., (1996), Verzhbitsky and Zolotarev (1989), and Rimi et al., (1998).*

Crustal Temperature Profiles compatible with Elevation, Geoid anomaly and Heat Flow



Comparisons illustrating the differences between the results of
Fullea et al (2007) and this work

Eastern Alboran Basin

Parameter	Fullea et al	This work	Difference	%
Moho Depth (km)	21.5	18.4	3.1	**14.5%**
Depth to base of Lithosphere (km)	86.2	64.2	22.0	**25.5%**
Moho Temperature	425.6	541.9	-116.3	**-27.3%**

Atlas Mountains

Parameter	Fullea et al	This work	Difference	%
Moho Depth (km)	35.3	31.3	4.0	**11.1%**
Depth to base of Lithosphere (km)	160.3	138.5	21.7	**13.6%**
Moho Temperature	399.4	500.1	-100.6	**-25.2%**

Study area of Alexandrino & Hamza, 2008
(State of Rio de Janeiro)

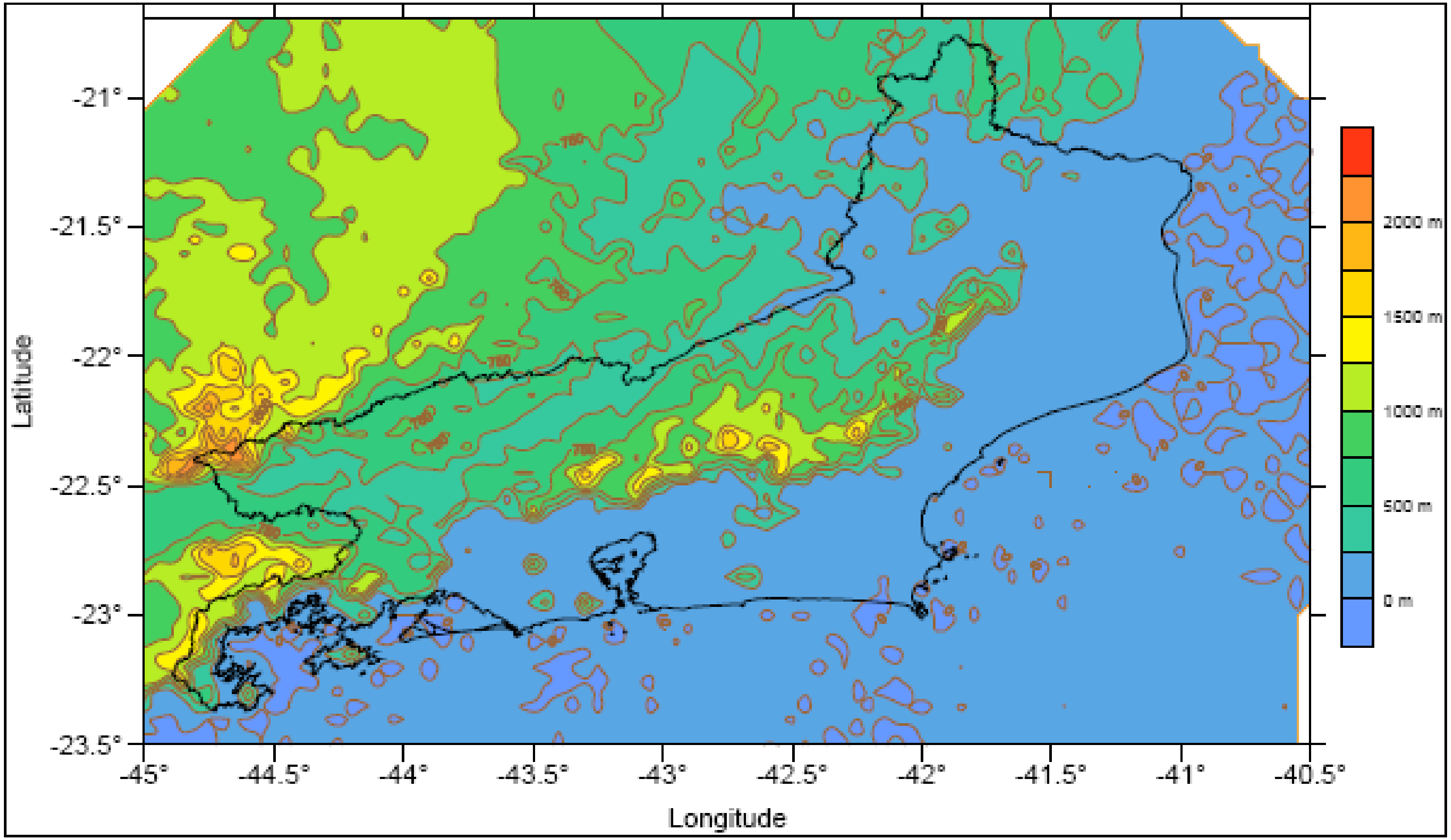


Figura 4: Modelo digital do terreno do Estado do Rio de Janeiro, com resolução de 12".

Digital Elevation Model, Escobar, 2006

Geoid Height (State of Rio de Janeiro)

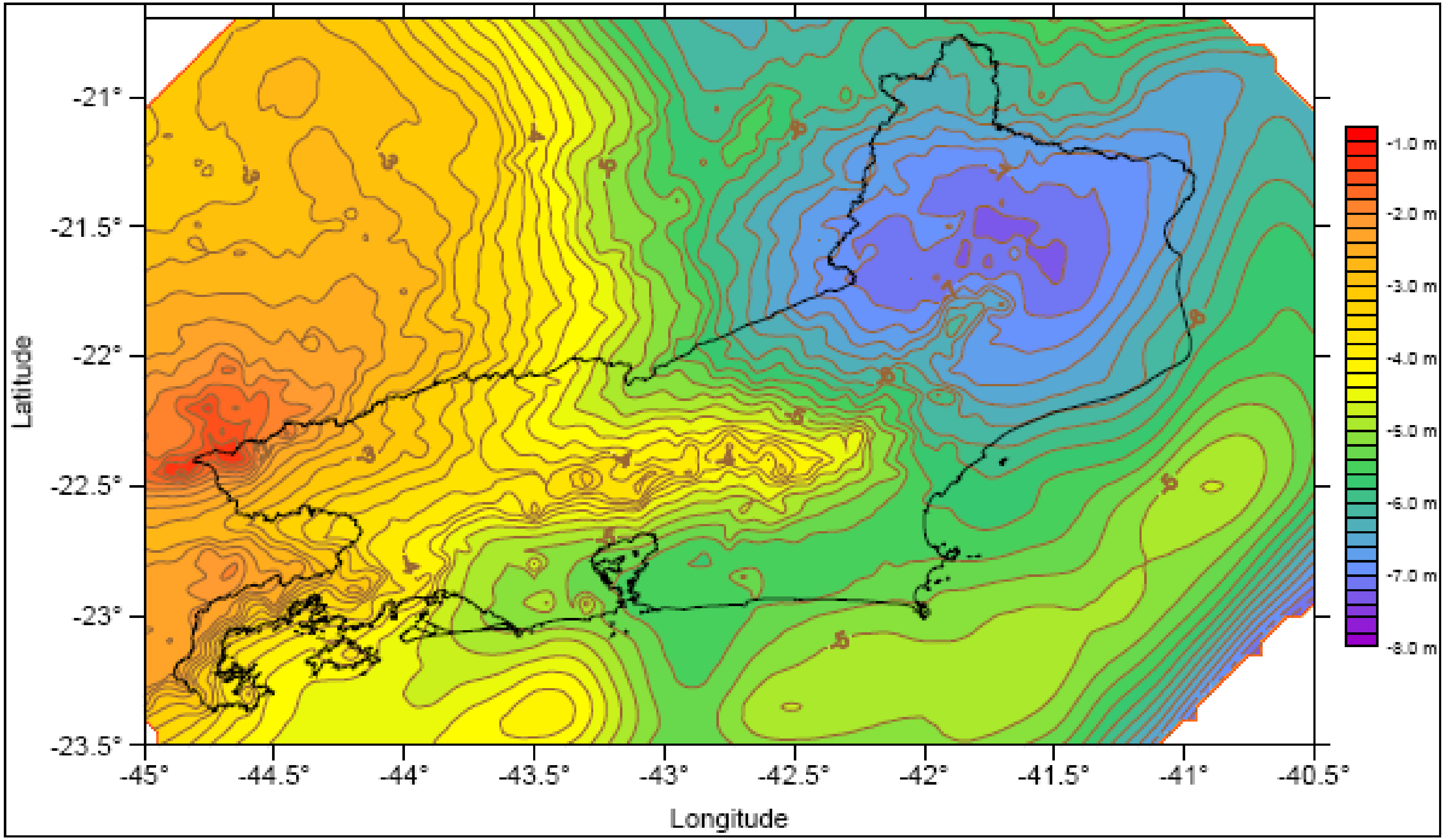
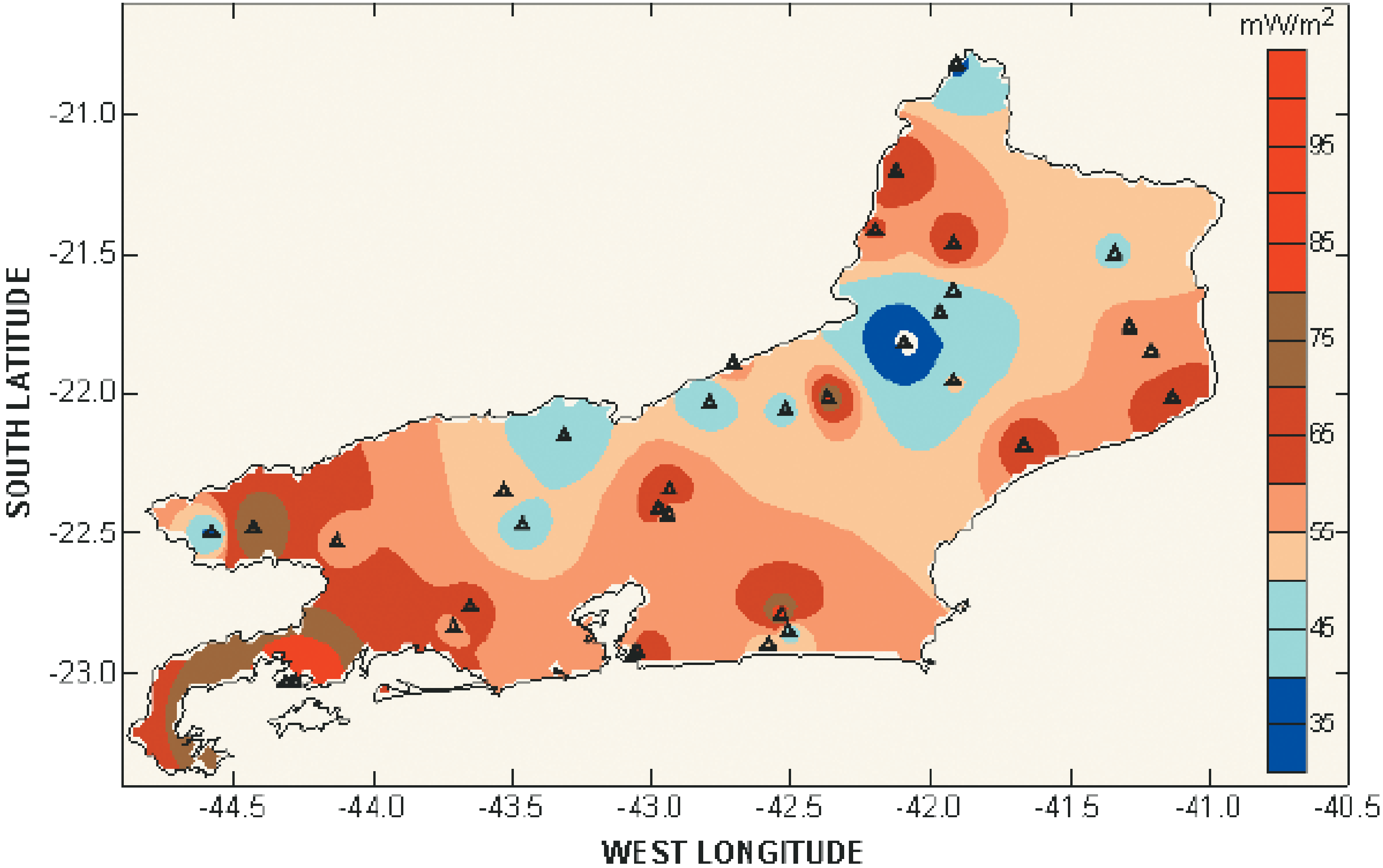


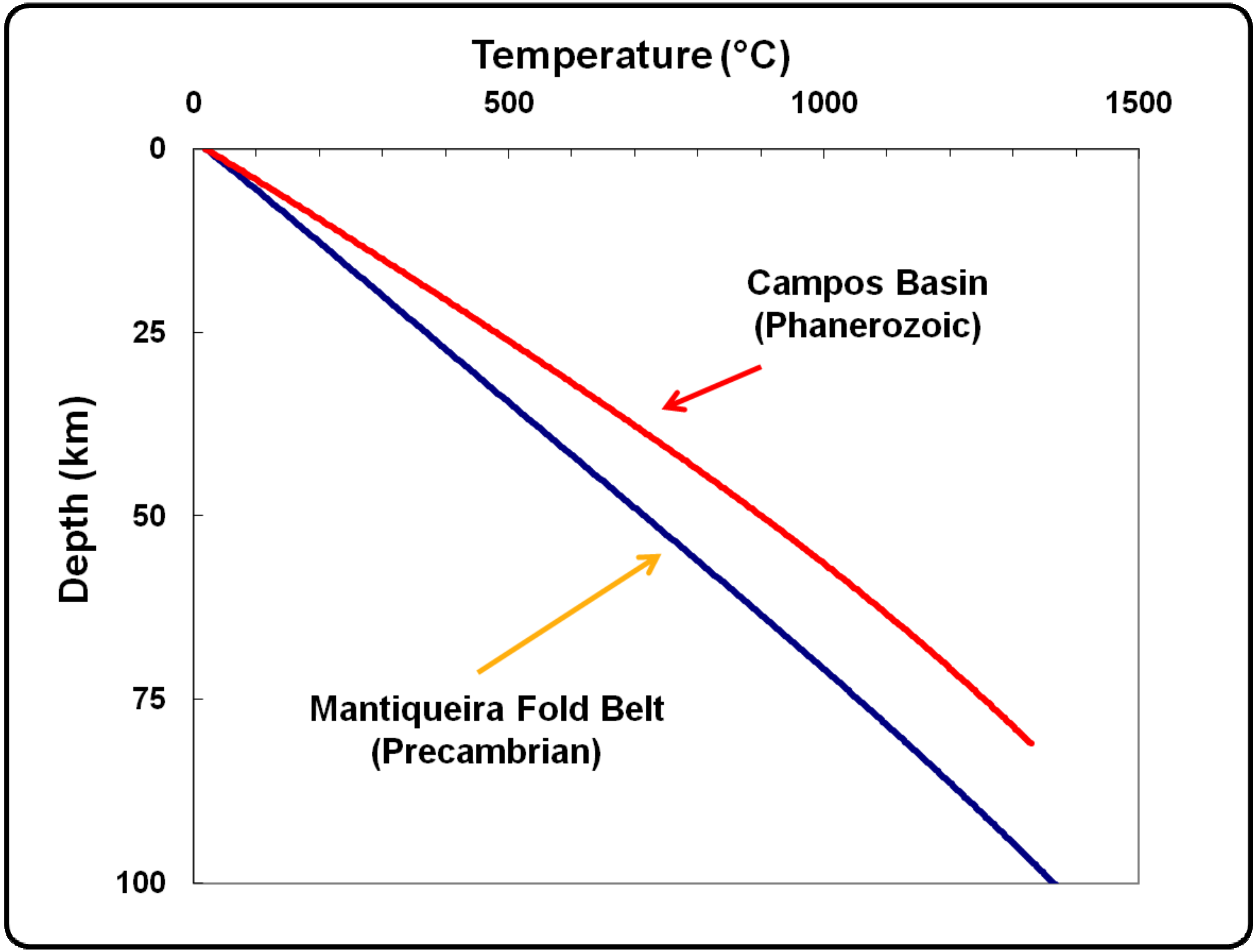
Figura 10: Alturas geoidais do Estado do Rio de Janeiro. Adição da componente geoidal regional (figura 2), componente geoidal residual (figura 8) e efeito indireto (figura 9).

Heat Flow Map (State of Rio de Janeiro)



(Gomes and Hamza, 2006)

Temperatures & Thermal Conductivity of crust in Passa Três (RJ), from joint inversion of elevation, geoide height and heat flow



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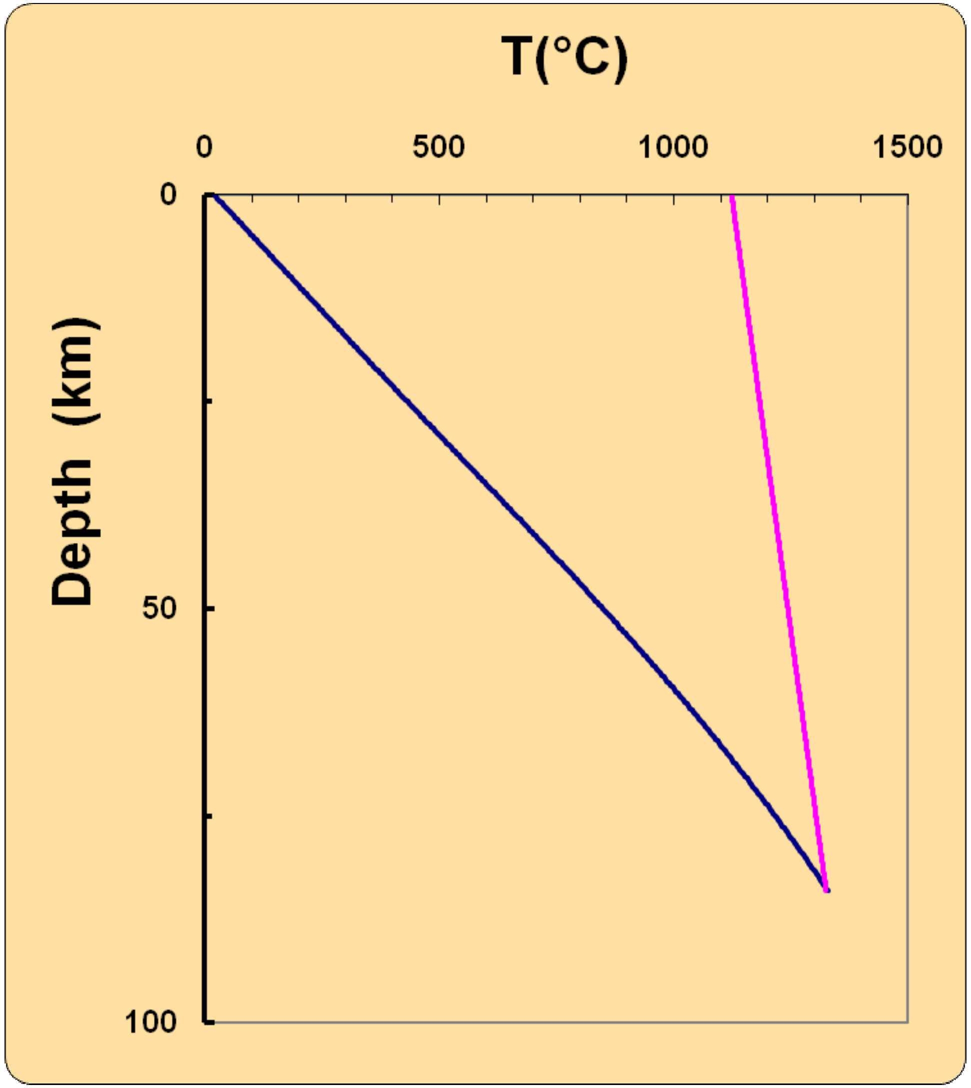
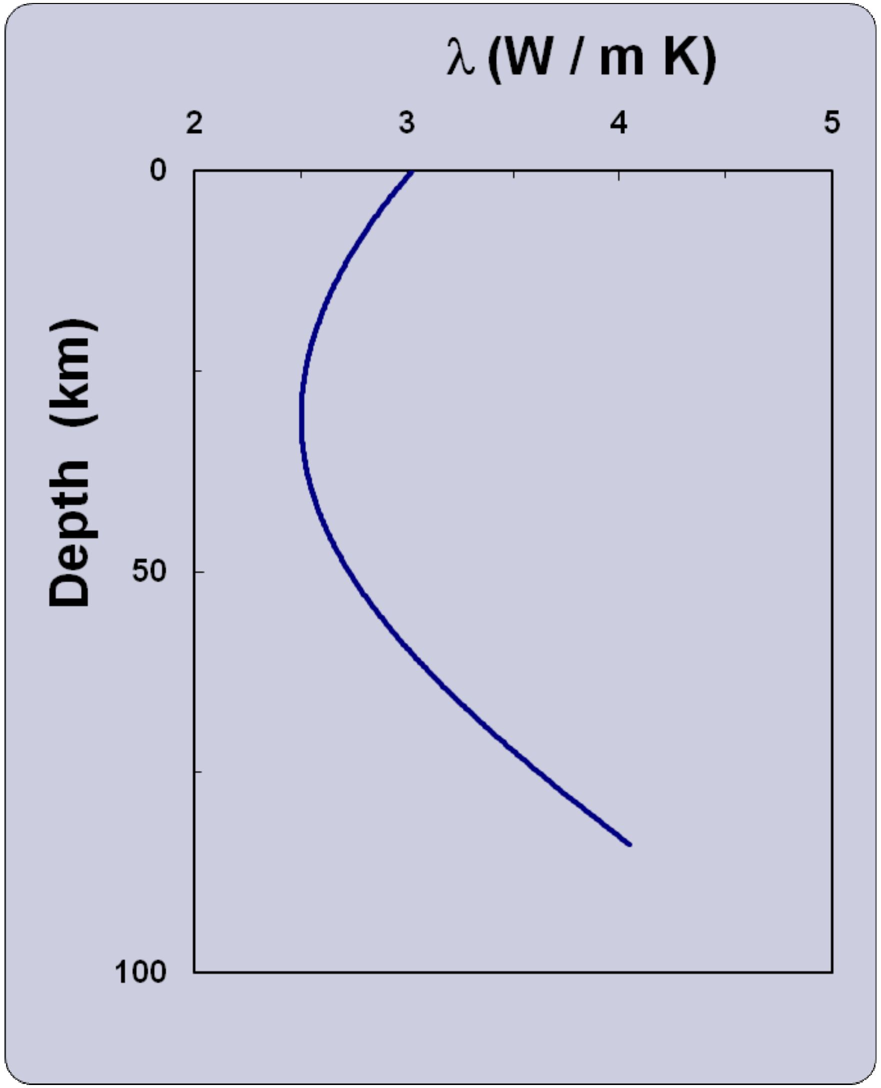




Conclusions

- The procedure proposed by Fullea et al (2007) allows joint inversion of elevation and geoid data but ignores the crucial role of surface heat flow in inversion. As a result the model leads to overestimation of crustal and lithospheric thickness and underestimation of moho temperatures;

- The modified method proposed in the present work allows simultaneous inversion of heat flow, elevation and geoid height. It takes into consideration vertical variations in thermal properties of the crust and provides information on the crustal temperature field at depth;

- The correlation between geoid height and heat flow is a useful tool for detailed mapping crustal temperature field in geothermal areas.

Thanks for your attention

Comparison of Lithosphere thickness (km)

Region	Fullea et al, 2007	This work
Alboran basin	100 - 110	50 - 70
Iberian Massif	100 - 120	80 - 100
Atlas Mountains	100 -140	100 -140

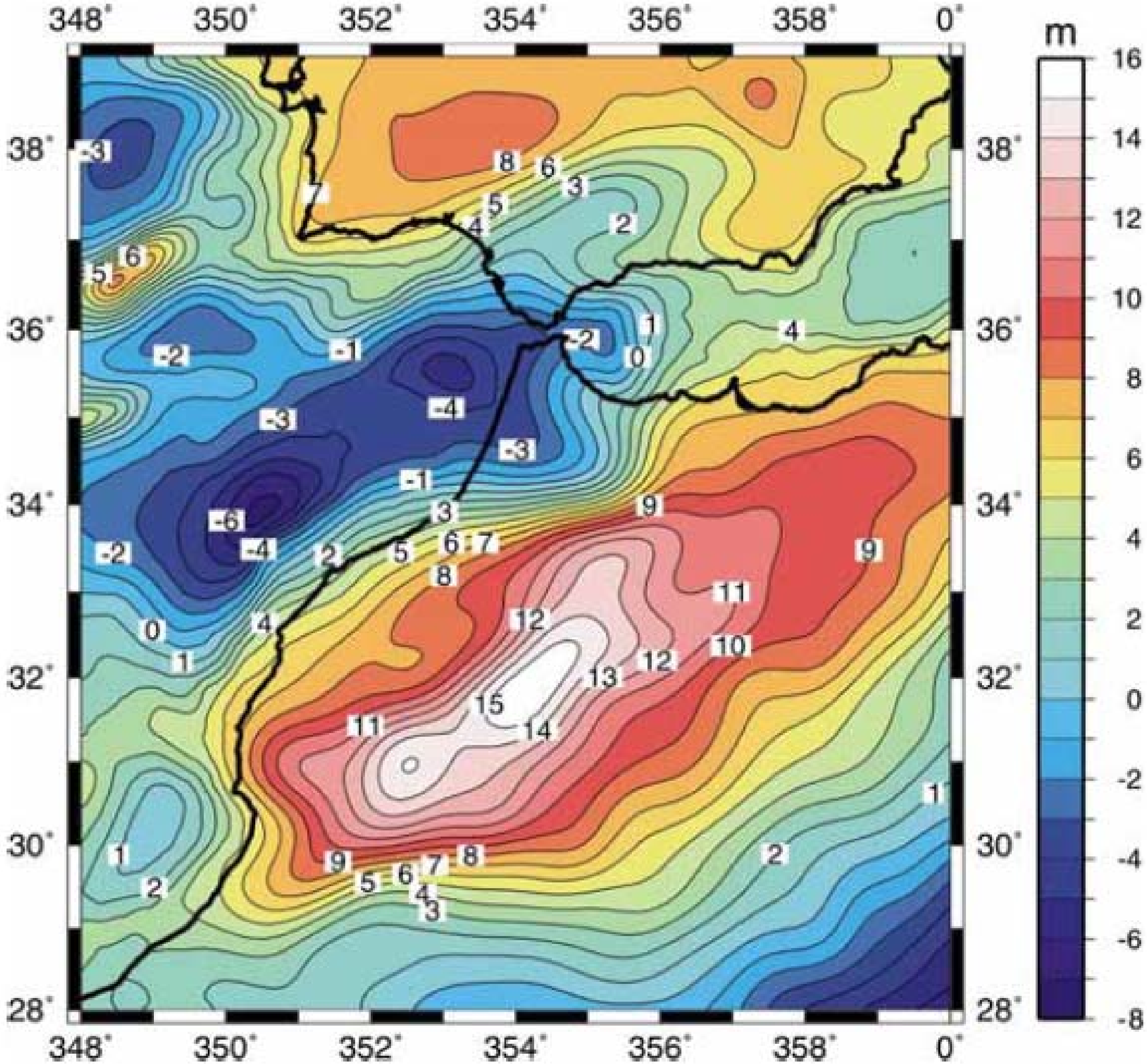


Fig. 3.3 *Geoid anomaly map from EGM96 Global Model (Lemoine et al., 1998). Long wavelengths (>4000 km) have been removed. Contour interval is 1 m.*

Consider a lithosphere composed of four layers:

Sea water, Crust, Lithospheric Mantle and Asthenosphere.

The relation between elevation and crustal thickness is:

$$z_c = \frac{\rho_a L_0 - E(\rho_a - \rho_w) + z_L(\rho_m - \rho_a)}{(\rho_m - \rho_c)}$$

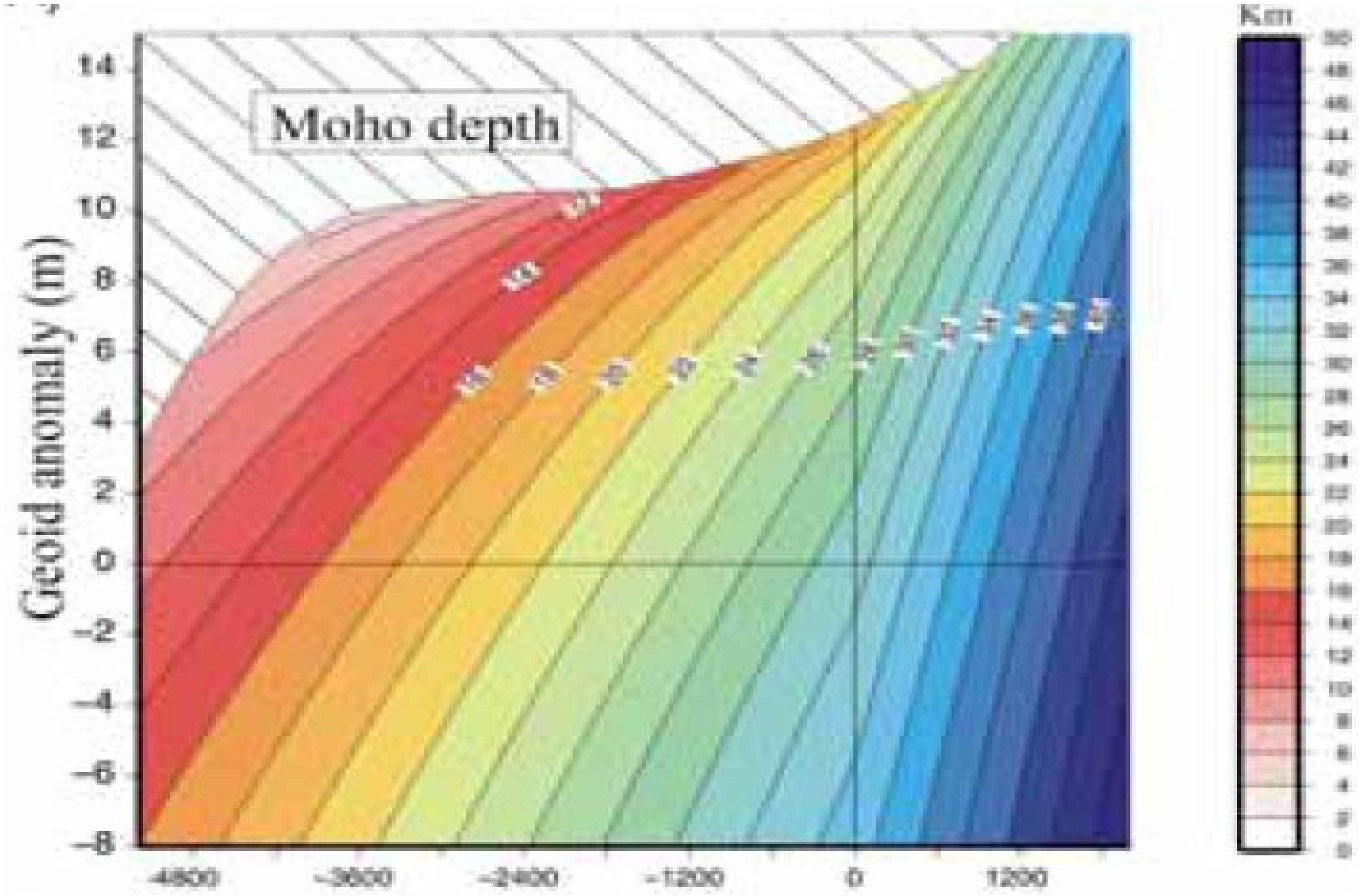
$$\rho_m(z) = \rho_a \left(1 + \alpha\left[T_a - T_m(z)\right]\right)$$

The temperature at the base of the crust may be expressed using the known value of surface heat flow or calculated value of basal heat flow (Modified after Fullea et al, 2007):

$$T_{mh} = T_s + \frac{q_m}{k_c}(z_c + E) - \frac{f}{k_c}$$

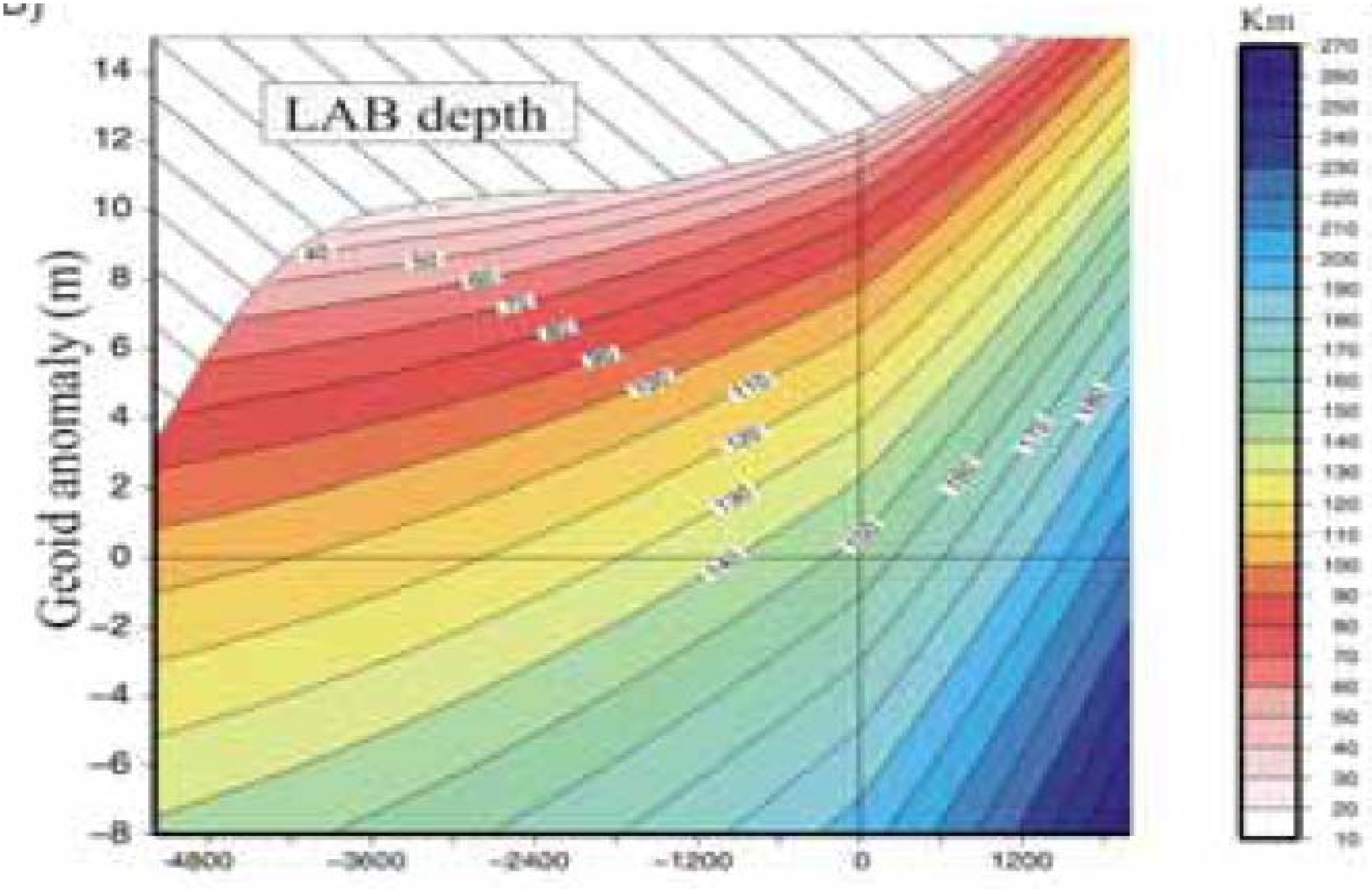
$$T_{mh}(E, z_c, z_L) = \frac{(z_L - z_c)\theta + \delta}{z_c \Delta k + z_L k_c + E k_m}$$

Variation of Moho in geoide height – elevation domain
(thinner crust means loss of buoyancy)



Elevation (m)

Variation of Moho in geoide height – elevation domain
(Thicker lithosphere leads to gain of buoyancy)



Elevation (m)

1 – Input data Modules

Fullea et al, 2007

Moho Temperature	
Equation 10	
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delta	1,1860E+08
deltaK	0,70
T_{Moho}	**511,45**

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Moho Temperature	
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Param B	6.82E-04
Param C	6.32E-10
Heat flow$_{Surface}$	5.00E-02
Moho Heat Flux	4.21E-02
Conductivity	3.00
T_{Moho}	**451.32**

2 – Modules of Iterative Processes

Fullea et al, 2007

Initial Estimates		
Moho Depth (km)	zc ref	26,95
Lithosphere Thickness (km)	zL ref	91,30

Alexandrino & Hamza, 2008

Double Iteration Process		
Moho Depth (km)	zc ref	26.23
Lithosphere Thickness (km)	zL ref	83.80
Feedback of Z_L for Temperature		83.88